Decentralized Formation Control of Multiple Autonomous Underwater Vehicles with Input Saturation Using RISE Feedback Method

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Abstract—Decentralized formation control for multiple autonomous underwater vehicles (AUVs) with input saturation is discussed in this paper. In the assumed group communication, each AUV can only obtain information from its adjacent neighbors in the specified communication topology, and only a subset of the AUVs can access the desired trajectory. A robust integral of the sign of the error (RISE) feedback based distributed controller with an anti-windup compensator is proposed so that all group members ultimately synchronize and achieve the desired formation. Based on Lyapunov theory, it is proven that the formation error of each AUV converges to zero. Finally, a simulation example demonstrates the effectiveness of the algorithm.

Index Terms—formation control, RISE feedback, anti-windup

I. INTRODUCTION

Autonomous underwater vehicles (AUVs) play an important role in ocean exploration. For complex missions, it may be necessary to employ multiple AUVs to improve the efficiency and the robustness of the system. Numerous approaches for controlling multi-agent systems (MAS) have been proposed and mainly focus on two topics: cooperative regulation control and cooperative tracking control. For cooperative regulation control, the main task is to design a control law that allows the system to reach an agreement or consensus equilibrium satisfying certain desired properties [1], [2], [3]. As for cooperative tracking control, it is the consensus of a group of agents with an active or virtual leader, whose motion is independent of all the other agents. The leader's information is only available to a portion of the agents of the MAS and thus the followers track the trajectory that the leader generates [4], [5], [6].

Formation control, which is one of the topics addressed in cooperative tracking control, generally aims to drive multiple vehicles to prescribed constraints on their states. Making various assumptions regarding the sensing capabilities and the interaction topology of the agents, a variety of formation control problems have been studied in the literature. Formation control problems for UAV swarm systems to achieve time-varying formations were studied in [7], which employed consensus approaches in its solution. By using attractive forces toward a circle, a control strategy for regular polygon formations of mobile robots was proposed based on their

virtual structure in [8]. A formation control method based on inter-agent displacements for single-integrator modeled agents in the plane was demonstrated in [9]. The key feature of that proposed control strategy is that it utilizes only distance and relative-angle measurements. A novel fault tolerant leader-follower formation control scheme for a group of underactuated autonomous surface vessels with partially known control input gain functions is proposed in [10].

The problem of multiple AUV formation control is well-known and challenging, not only because the multiple AUVs may have highly complex nonlinear dynamics and coupling, but also because of the uncertain, dynamical, and adversarial underwater environments in which they may operate. A variety of methods have addressed multiple AUV formation control problems [11], [12], [13], [14]. In spite of the rich literature in this field, there still exist several critical issues that have not been adequately addressed. In particular, virtually all of the multi-AUV formation control algorithms from the above-mentioned references are developed under a strong assumption that all individual AUV agents in the group have identical (homogenous) system dynamics and precise mathematical models for the AUVs are available for distributed control design. This assumption may be unrealistic in practical AUV formation control applications.

In this paper, decentralized formation control for multiple AUVs with input saturation is discussed. The main features of this paper are as follows.

1) The robust integral of sign of the error (RISE) feedback control algorithm [15], [16], [17] is employed to accomodate unknown dynamics of the vehicles and the disturbances of the external environment. Compared with a traditional PID method, RISE feedback is more robust, as the integral of the signum function has been added to compensate for model uncertainty and unknown dynamics of the system and make the input continuous.

2) An anti-windup compensator is proposed to aid the controller in mitigating input saturation. The RISE algorithm is viewed as a high gain strategy, and so our system is confronted with the risk of demanding large control inputs which can reach outside the range of actuation values. So a saturated controller is added to the input, and its influence on the system will be taken into account.

3) Each AUV can only achieve its adjacent neighbours'

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information (according to the specified communication topology) and only a subset of the AUVs can access the desired trajectory. A virtual leader is introduced to the fixed directed topology.

The rest of the paper is organized as follows. In Section II, some preliminaries are given briefly. In Section III, the RISE feedback decentralized formation control law is proposed in terms of input saturation. A simulation is used to demonstrate the effectiveness of the control scheme in Section IV.

II. PROBLEM FORMULATION

In this paper, a weighted directed graph is used to model information exchange among the agents. In addition, we introduce a virtual agent, represented by v_0, and it can obtain the desired trajectory explicitly. Then for the N agent network system, its graph $\mathcal{G}$ contains a node set $\mathcal{V} = \{v_0, v_1, \ldots, v_N\}$, and a weighted adjacency matrix $A^* = [a^*_{ij}] \in \mathbb{R}^{(N+1)\times(N+1)}$, where $a^*_{ij} > 0$ indicates that agent i can receive the information from agent j, otherwise $a_{ij} = 0$. Define a diagonal matrix $\Delta(\mathcal{G}) \in \mathbb{R}^{(N+1)\times(N+1)}$ with elements $\delta_{ii} = \sum_k a_{ik}$, and the normalized Laplacian of $\mathcal{G}$ as $L = I - A$, where the elements in the normalized adjacency matrix A are defined as $a_{ij} = a^*_{ij}/\delta_{ii}$, $\delta_{ii} \neq 0$, and $a_{ij} = a^*_{ij}, \delta_{ii} = 0$. By adding a virtual agent in the system, we call the graph $\mathcal{G}$ the *extended communication graph*. For each agent let $\mathcal{N}_i = \{v_j \in \mathcal{V} | a_{ij} > 0\}$ denote the neighbor set of v_i.

Assumption 1: The extended communication graph $\mathcal{G}$ has a spanning tree with the virtual agent as the root, and this virtual agent follows the desired trajectory strictly.

From [18] we know the Laplacian matrix of the weighted directed graph is a positive definite matrix. This property will be used for the following stability analysis.

Let us consider a system composed of N AUVs (numbered 1, 2, ..., N). The kinematic model and dynamic model of the ith AUV are as follows:

$$\dot{\eta}_i = J(\eta_i)\nu_i$$
$$M_i\dot{\nu}_i + C_i(\nu_i)\nu_i + D(\nu_i)\nu_i + g_i(\eta_i) + d_i = \tau_i(t), \quad (1)$$

where $\eta_i = [\eta^T_{1i}, \eta^T_{2i}]^T$, $\nu_i = [\nu^T_{1i}, \nu^T_{2i}]^T$. $\eta_{1i} = [x_i, y_i, z_i]^T$ is the position vector and $\eta_{2i} = [\phi_i, \theta_i, \psi_i]^T$ is the orientation vector defined in the earth-fixed frame. $\nu_{1i} = [u_i, v_i, w_i]^T$ is the linear velocity and $\nu_{2i} = [p_i, q_i, r_i]^T$ is the angular velocity defined in the body-fixed frame, $M_i, C_i, D_i \in \mathbb{R}^{6*6}$ and $g_i \in \mathbb{R}^6$ are the inertial matrix, Coriolis matrix, damping matrix and restoring forces and moment vector, and $d_i \in \mathbb{R}^6$ is the external disturbance. $J(\eta_i) \in \mathbb{R}^{6*6}$ is the rotational transformation matrix. We transform the body-fixed dynamic model of (1) into earth-fixed coordinates, and then we obtain:

$$M^*_i(\eta_i)\ddot{\eta}_i + C^*_i(\dot{\eta}_i, \eta_i)\dot{\eta}_i + D^*_i(\dot{\eta}_i, \eta_i)\dot{\eta}_i + g^*_i(\eta_i) + d^*_i = \tau_{ni}, \quad (2)$$

where the terms of Equation (2) are defined as follows:

$$M^*(\eta)_i = J^{-T}(\eta)M_i J^{-1}(\eta)$$
$$C^*_i(\dot{\eta}, \eta = J^{-T}(\eta)[C_i(\dot{\eta}) - M_i J^{-1}(\eta)\dot{J}(\eta)]J^{-1}(\eta)$$
$$D^*_i(\dot{\eta}, \eta) = J^{-T}(\eta)D_i(\dot{\eta})J^{-1}(\eta)$$
$$g^*_i(\eta) = J^{-T}(\eta)g_i(\eta) \quad (3)$$
$$d^*_i = J^{-T}(\eta)d_i$$
$$\tau^*_i = J^{-T}(\eta)\tau_i.$$

We give several assumptions about the dynamics of (1):

1) The inertial matrices $M^*_i(\eta)$ are symmetric and positive. Thus, the following is satisfied:

$$m_1||y||^2 \leq y^T M^*_i(\eta)y \leq \bar{m}(\eta)||y||^2, \quad (4)$$

where $m_1 \in \mathbb{R}^6$ are known positive constants, $\bar{m}(\eta) \in \mathbb{R}$ are known positive functions and $y \in \mathbb{R}^6$ are bounded vectors.
2) If $\eta(t)$ and $\dot{\eta} \in \mathcal{L}_\infty$, then $C(\eta, \dot{\eta})$ and $D(\eta, \dot{\eta})$ are bounded.
3) The disturbance $d_i(t)$, $\dot{d}_i(t)$ and $\ddot{d}_i(t)$ are all existing and bounded.

For successful decentralised formation control of multiple AUVs, the tracking error and the relative error of each neighbouring AUV must converge to a compact set. Two kinds of errors are defined as follows:

1) The tracking error between the ith AUV and the desired trajectory is:

$$e_i = \eta_{id} - \eta_i, i = 1, 2, \ldots, N. \quad (5)$$

2) The relative position error of the ith AUV and its neighbour jth AUV is:

$$e_{ij} = (\eta_i - \eta_j)$$
$$= (\eta_{id} - \eta_{jd}) - (\eta_i - \eta_j) \quad (6)$$
$$= e_i - e_j, i, j = 1, 2, \ldots, N.$$

The problem we discuss in this thesis can now be formulated. For the synchronization tracking control of a multiple AUV system, a distributed control algorithm should be designed to reach the following goals:

$$\lim_{t\to\infty}[\eta_i(t) - \eta_{id}(t)] = 0, i = 1, 2, \ldots, N$$
$$\lim_{t\to\infty}[e_i(t) - e_j(t)] = 0, i = 1, 2, \ldots, N, \quad (7)$$

under the condition that only a subset of the AUVs can access the desired trajectory and each AUV can only sense its own and its neighbours' state information.

III. CONTROL LAW DESIGN

We next introduce the synchronization error:

$$s_i = \sum_{j\in\mathcal{N}_i}(e_i - e_j), i = 1, 2, \ldots, N, \quad (8)$$

and the auxiliary error:

$$\epsilon_i = \dot{s}_i + \lambda_{1i}s_i$$
$$\varsigma_i = \dot{\epsilon}_i + \lambda_{2i}\epsilon_i, \tag{9}$$

where $\epsilon_i \in \mathbb{R}^6$ and $\varsigma_i \in \mathbb{R}^6$, and $\lambda_{1i}, \lambda_{2i}$ are positive constants. We assume that the $\ddot{\eta}_i$ cannot be measured, so the system cannot obtain the value of ς_i directly.

With (9) multiplying $M_i^*(\eta)$, we have

$$M_i^*\varsigma_i = M_i^*(\lambda_{1i}\dot{s}_i + \lambda_{2i}\epsilon_i) + M_i^*\ddot{s}_i, \tag{10}$$

and then we express Equation (10) in matrix form:

$$M^*\varsigma = M^*(\lambda_1\dot{s} + \lambda_2\epsilon) + M^*L \otimes I_N\ddot{e}, \tag{11}$$

where:

$$M^* = \text{diag}[M^*(\eta_1), M^*(\eta_2), \ldots, M^*(\eta_N)]$$
$$\varsigma = [\varsigma_1^T, \varsigma_2^T, \ldots, \varsigma_N^T]^T$$
$$\alpha_1 = \text{diag}[\alpha_{11}, \alpha_{12}, \ldots, \alpha_{1N}]$$
$$s = [s_1^T, s_2^T, \ldots, s_N^T] \tag{12}$$
$$\alpha_2 = \text{diag}[\alpha_{21}, \alpha_{22}, \ldots, \alpha_{2N}]$$
$$\epsilon = [\epsilon_1^T, \epsilon_2^T, \ldots, \epsilon_N^T]^T$$
$$\ddot{e} = [\ddot{e}_1^T, \ddot{e}_2^T, \ldots, \ddot{e}_N^T]^T,$$

L is the Laplacian matrix of the system toplogy, and I_N are $N \times N$ identity matrices. It can be concluded from (2) that

$$\ddot{\eta} = M^{*-1}(\ddot{\eta})[\tau_n - C^*(\dot{\eta}, \eta)\dot{\eta} - D^*(\dot{\eta}, \eta)\dot{\eta} - g^*(\eta) - \tau_d^*], \tag{13}$$

where:

$$\eta = [\eta_1^T, \eta_2^T, \ldots, \eta_N^T]^T$$
$$M^{*-1} = \text{diag}\{M^{*-1}(\eta_1), M^{*-1}(\eta_2), \ldots, M^{*-1}(\eta_N)\}$$
$$C^* = \text{diag}\{C^*(\dot{\eta}_1, \eta_1), C^*(\dot{\eta}_2, \eta_2), \ldots, C^*(\dot{\eta}_N, \eta_N)\}$$
$$D^* = \text{diag}\{D^*(\dot{\eta}_1, \eta_1), D^*(\dot{\eta}_2, \eta_2), \ldots, D^*(\dot{\eta}_N, \eta_N)\} \tag{14}$$
$$g^* = [g^{*T}(\eta_1), g^{*T}(\eta_2), \ldots, g^{*T}(\eta_N)]^T$$
$$\tau_d^* = [\tau_{d1}^{*T}, \tau_{d2}^{*T}, \ldots, \tau_{dN}^{*T}]^T$$
$$\tau_n = [\tau_{n1}^T, \tau_{n2}^T, \ldots, \tau_{nN}^T]^T.$$

Combining Equations (9) and (13) into (10), we have

$$M^*\varsigma = M^*(\lambda_1\dot{s} + \lambda_2\epsilon)$$
$$- M^*L \otimes I_N M^{*-1}\tau^*$$
$$+ M^*L \otimes I_N[\ddot{\eta}_d$$
$$+ M^{*-1}(C^*\dot{\eta} + D^*\dot{\eta} + g^* + d^*)].$$

Next, we let:

$$P = M^*[\alpha_1\dot{s} + \alpha_2\epsilon + L \otimes I_N(\ddot{\eta}_d$$
$$- M^{*-1}(C^*\dot{\eta} + D^*\dot{\eta} + g^*))] - F_d, \tag{15}$$

where:

$$F_d = M^*(\eta_d)\ddot{\eta}_d + C^*\dot{\eta}_d$$
$$+ D^*\dot{\eta}_d + g^*(\eta_d) + M^*[L \otimes I_N]M^{*-1}\tau_d^*.$$

Then, Equation (15) can be simplified into:

$$M^*\varsigma = P + F_d - M^*L \otimes I_N M^{*-1}\tau^*. \tag{16}$$

Incorporating (15), we then have:

$$M^*\dot{\varsigma} = -\frac{1}{2}\dot{M}^*\varsigma - \epsilon + \Sigma_d$$
$$+ \Sigma - M^*L \otimes I_N M^{*-1}\dot{\tau}^*, \tag{17}$$

where:

$$\Sigma_d = \dot{F}_d$$
$$\Sigma = -\frac{1}{2}\dot{M}^*\varsigma + \epsilon + \dot{P}. \tag{18}$$

It is known from the property of the model that Σ is bounded. Defining the vector $Z = [s^T, \epsilon^T, \varsigma^T]^T$, we may then state the following:

$$\Sigma \leq \Pi(||Z||)||Z||, \tag{19}$$

where $\Pi(||Z||)$ is an increasing function about $||Z||$, and we still obtain the same result that:

$$||\Sigma_d|| \leq \Xi_1, ||\dot{\Sigma}_d|| \leq \Xi_2. \tag{20}$$

Proposing the following control law:

$$\tau_i^*(t) = (K_i + I_3)\epsilon_i(t)$$
$$+ \int_0^t [(K_i + I)\alpha_{1i}\epsilon_i(\sigma) + \Lambda_i\text{sgn}(\epsilon_i(\sigma))]d\sigma, \tag{21}$$

where K_i and Λ_i are diagonal positive gain matrices, $\text{sgn}(\epsilon_i(\sigma)) = [\text{sgn}(\epsilon_{i1}(\sigma)), \text{sgn}(\epsilon_{i2}(\sigma)), \text{sgn}(\epsilon_{i3}(\sigma))]^T$.

As the controller (21) is considered to be a high gain strategy, the computed input values potentially exceed our AUVs' actuation limits. So we introduce the saturation of the input to decrease that risk. In addition, an anti-windup compensator is exploited to reduce the effects of saturation.

The input saturation is described as follows:

$$\tau_i^* = \text{sat}(\tau_{\text{ir}}), \tag{22}$$

where τ_{ir} is the actual control input, which is the output of the saturated actuation, denoted as

$$\tau_i^* = \text{sat}(\tau_{\text{ir}}) = \begin{cases} \tau_m, & \tau_i^* > \tau_m \\ \tau_{ir}, & -\tau_m \leq \tau_i^* \leq \tau_m \\ -\tau_m, & \tau_i^* < -\tau_m \end{cases} \tag{23}$$

where τ_m is the known bound of the actuator saturation. In order to reduce the effects of saturation, an anti-windup compensator is introduced into the control law. The approach for the complete system is illustrated in Fig.1.

The anti-windup compensator is designed as follows:

$$\dot{w}_i = \begin{cases} -\beta_{1i}w_i - \frac{|\beta_{2i}s_i^T\triangle\tau_i| + \frac{1}{2}\triangle\tau_i^T\triangle\tau_i}{w_i^T w_i}w_i + \triangle\tau_i, & ||w|| \geq \iota \\ 0, & ||w|| < \iota \end{cases} \tag{24}$$

where $\triangle\tau_i = \tau_{ir} - \tau_i^*$, w_i is the state of the auxiliary system, $\beta_{1i}, \beta_{2i} \in \mathbb{R}^{6*6} > 0$ are the designed parameters, and ι is a small positive constant.
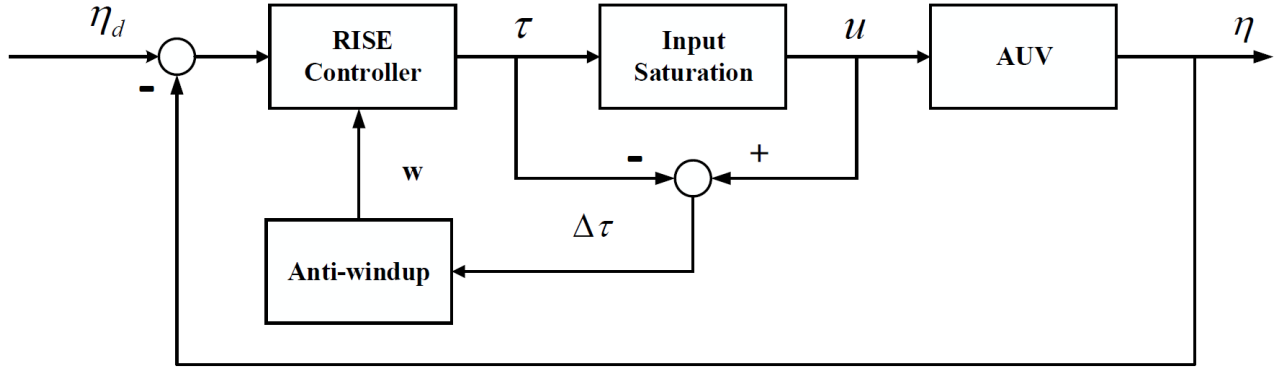
Fig. 1: Structure of the proposed control system.

Considering the actuator saturation, the control law is proposed as:

$$\tau_i = (K_i + 1)\epsilon_i(t)$$
$$+ \int_0^t [(K_i + 1)\alpha_{1i}\epsilon_i(\sigma) - (K_i + 1)w_i \quad (25)$$
$$+ \Lambda_i \text{sgn}(\epsilon_i(\sigma))]d\sigma.$$

Theorem 1: We consider a group of AUVs. Their communication graph contains a spanning tree and the virtual AUV is the root. Each AUV can only obtain its neighbours' information and a subset of the AUVs can access the desired trajectory. We assume the proposed control law (25) is implemented. When the time $t \to \infty$, the formation error $s_i(t) \to 0$, $\epsilon_i(t) \to 0$, for $i = 1, 2, ..., N$.

Proof: Transform (21) into a matrix set, and we may express it as:

$$\dot{\tau} = (K_i + I) \otimes I_N \varsigma + \Lambda_i \otimes I_N \text{sgn}(\epsilon) - (K_i + I) \otimes I_N w, \quad (26)$$

where $\text{sgn}(\epsilon) = [\text{sgn}(\epsilon_1)^T, \text{sgn}(\epsilon_2)^T, \ldots, \ldots, \text{sgn}(\epsilon_N)^T]^T$.

Consider the following Lyapunov candidate function,

$$V = \frac{1}{2}s^T s + \frac{1}{2}\epsilon^T \epsilon + \frac{1}{2}\varsigma^T M^* \varsigma + \frac{1}{2}w^T w + \Delta, \quad (27)$$

where:

$$\Delta = \Xi_1 - \int_{t_0}^t \Psi(\tau)d\tau, \quad (28)$$

and the following inequality holds for Ψ:

$$\int_{t_0}^t \Psi(\tau)d\tau \leq \Xi_1. \quad (29)$$

From the definition of V it is clear that the function V is positive and bounded. From (27), we may derive the following:

$$\dot{V} = s^T \dot{s} + \epsilon^T \dot{\epsilon} + \varsigma^T(M^*\dot{\varsigma} + \frac{1}{2}\dot{M}^*) + \dot{\Delta}$$
$$= s^T(\epsilon - \alpha_1 s) + \epsilon^T(\varsigma - \alpha_2 \epsilon)$$
$$+ \varsigma^T \{-\epsilon + \Sigma_d + \Sigma - M^* L \otimes I_N M^{*-1} *$$
$$[(K_i + I) \otimes I_N \varsigma + \Lambda_i \otimes I_N \text{sgn}(\epsilon) - (K_i + I) \otimes I_N w]\}$$
$$+ w^T \dot{w} + \dot{\Delta}. \quad (30)$$

From the property of the Laplacian matrix, we know that $H = L \otimes I_N$ is Hurwitz, and so is the matrix $K = (K_i + I) \otimes I_N$. Let the minimum eigenvalues of H and K be $\lambda_{\min}(H) > 0$, and $\lambda_{\min}(K) > 1$. Then we have:

$$\dot{V} \leq -[\lambda_{\min}(\alpha_1) - \frac{1}{2}]||s||^2 - [\lambda_{\min}(\alpha_2) - \frac{1}{2}]||\epsilon||^2$$
$$+ \varsigma^T \{\Sigma_d + \Sigma - \lambda_{\min}(H)[(K_i + I) \otimes I_N \varsigma \quad (31)$$
$$+ \Lambda_i \otimes I_N \text{sgn}(\epsilon) - (K_i + I) \otimes I_N w]\} + \dot{\Delta}$$
$$- (\beta_1 w^T w + |\beta_2 s^T \triangle \tau| + \frac{1}{2}\triangle \tau^T \triangle \tau + w^T \triangle \tau).$$

We combine (19) and (28) to obtain:

$$\Psi = \varsigma^T [\Sigma_d - \lambda_{\min}(H)\Lambda_i \otimes I_N \text{sgn}(\epsilon)], \quad (32)$$

and then finally we may state:

$$\dot{V} \leq -\rho||Z||^2 - \frac{1}{2}\lambda_{\min}(H)\lambda_{\min}(K)||\varsigma||^2 + ||\varsigma||\Pi(||Z||)||Z||$$
$$- [||\beta_1|| - \frac{1}{2} - \frac{1}{2}\lambda_{\min}(H)\lambda_{\min}(K)]||w||$$
$$\leq -[\rho - \frac{\Pi^2(||Z||)}{2\lambda_{\min}(H)\lambda_{\min}(K)}]||Z||^2$$
$$- [||\beta_1|| - \frac{1}{2} - \frac{1}{2}\lambda_{\min}(H)\lambda_{\min}(K)]||w||, \quad (33)$$

where $\rho = \min\{\lambda_{\min}(\alpha_1) - \frac{1}{2}, \lambda_{\min}(\alpha_2) - \frac{1}{2}\}$. It can be demonstrated that when $\rho - \frac{\Pi^2(||Z||)}{2\lambda_{\min}(H)\lambda_{\min}(K)} > 0$, and $||\beta_1|| - \frac{1}{2} - \frac{1}{2}\lambda_{\min}(H)\lambda_{\min}(K) > 0$, we then have $\dot{V} < 0$. From the stability properties of Lyapunov functions, it can be concluded that when $t \to \infty$, $||Z|| \to 0$, and at the same time, $\varsigma \to 0$, and $\epsilon \to 0$. From the definition of error and for the ith AUV, $\varsigma_i \to 0$, $\epsilon_i \to 0$, then $s_i \to 0$, $\eta_{id} - \eta_i \to 0$, and $(\eta_{id} - \eta_{jd}) - (\eta_i - \eta_j) \to 0$, for $i = 1, 2, \ldots, N$.

IV. SIMULATION

In this section, simulated results of formation control will be demonstrated by applying RISE feedback methods in conjunction with anti-windup compensation to a multi-AUV system. We assume that there are three AUVs in the topology, which are identical. Their communication graph is as illustrated in Fig. 2. The 0th node is the virtual leader and it generates the
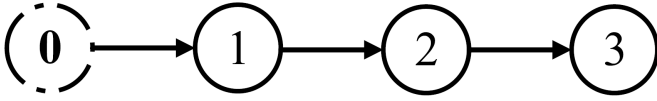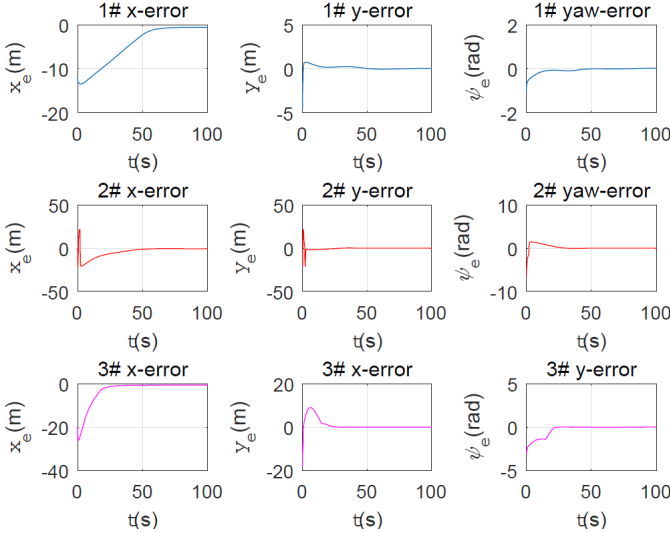
Fig. 2: Simulated multi-AUV communication toplogy.

Fig. 3: The tracking error of each AUV.

Fig. 4: The tracking velocity of each AUV.

Fig. 5: The control input of each AUV.

desired trajectory for the system. In this example, only the 1st node can achieve the information.

Only the kinematics and the dynamics of the vehicles in the horizontal plane are considered in the simulation. The hydrodynamic parameters M, C and D of each agent are $M = \mathrm{diag}\{25.8, 33.8, 2.76\}$, $D = \mathrm{diag}\{-12, -17, -0.5\}$, and $C = \{0, 0, -33.8v; 0, 0, 25.8u; 33.8v, -25.8u, 0\}$. The initial state for each agent is assumed to be $\eta_1 = [-10, 10, -\frac{\pi}{3}]^T$, $\nu_1 = [0.1, 0, 0]^T$, $\eta_2 = [-10, -10, -2\pi]^T$, $\nu_2 = [0.1, 0, 0]^T$, and $\eta_3 = [10, 10, \pi]^T$, $\nu_3 = [0.1, 0, 0]^T$. The desired tracking path is $x_d = t, y_d = 2\cos(0.1t)$. The limits of the actuation enforced in the simulation are ± 15N.

Simulation results are exhibited in Figs. 3 to 6. We find that good formation geometry performance is achieved and the tracking errors of all the AUVs converge to a small neighbourhood of zero, despite the face that some of the agents cannot access the desired trajectory directly and there exists input saturation.

V. CONCLUSION

In this paper, we have studied the problem of decentralized formation control with input saturation for multiple AUVs. Under the condition that the Laplacian matrix of the extended communication graph contains a spanning tree with the virtual agent as the root, a RISE feedback and anti-windup compensator-based formation control law has been designed for each agent. It has been shown that the tracking error of each agent converges to zero. Simulation results have shown the potential effectiveness of the proposed method.
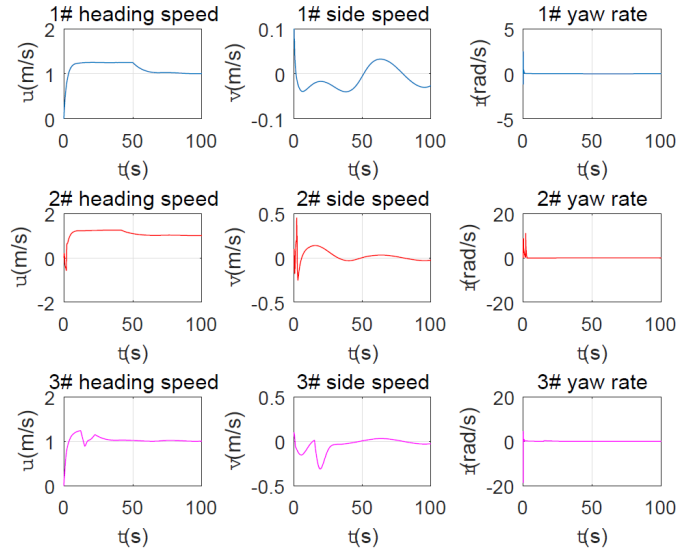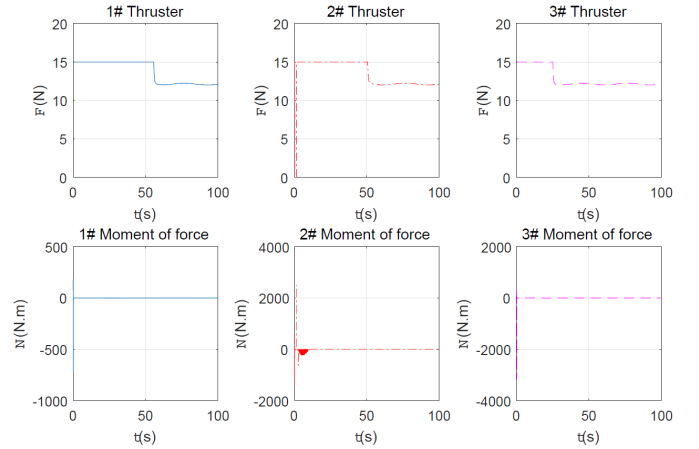
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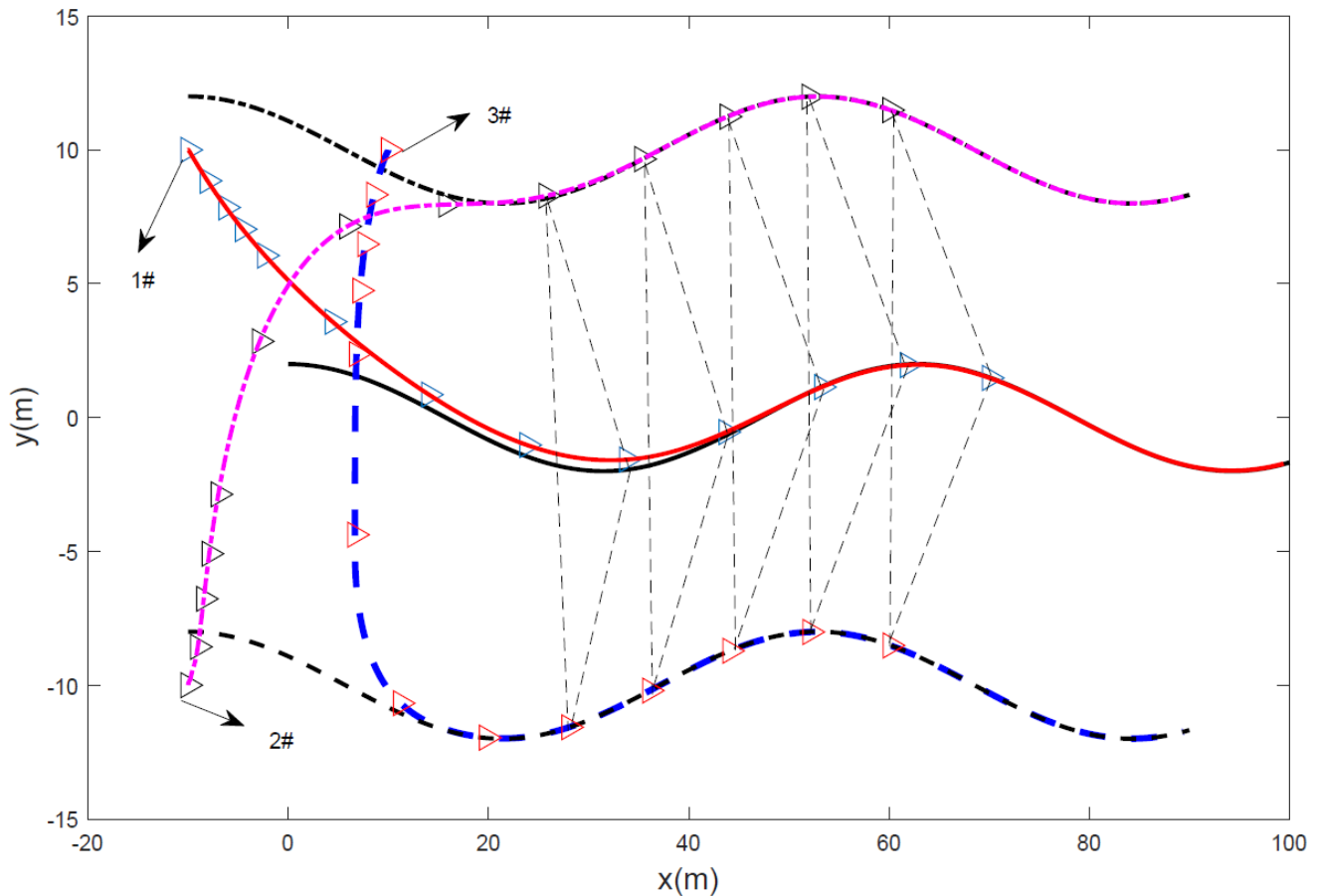
Fig. 6: The tracking path of the AUVs.

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