Ma 232 Linear Algebra **Darinka Dentcheva**
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Homework 2

(due Thursday, September 18, 2008)

Problem 1: Find $PA = L\Lambda U$ factorizations for the matrices:

$$\begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 2 & 3 & 4 \end{bmatrix} \quad \text{and} \quad \begin{bmatrix} 1 & 2 & 1 \\ 2 & 4 & 2 \\ 1 & 1 & 1 \end{bmatrix}$$

Problem 2: decide whether the following systems are singular or nonsingular, and whether they have no solution, one solution, or infinitely many solutions:

$$\begin{aligned} v - w &= 2 \\ u - v &= 2 \\ u - w &= 2 \end{aligned} \quad \text{and} \quad \begin{aligned} v - w &= 0 \\ u - v &= 0 \\ u - w &= 0 \end{aligned} \quad \text{and} \quad \begin{aligned} v + w &= 1 \\ u + v &= 1 \\ u + w &= 1 \end{aligned}$$

Problem 3: What three elimination matrices E_{21}, E_{31}, E_{32} put the matrix

$$A = \begin{bmatrix} 1 & 0 & 1 \\ 2 & 2 & 2 \\ 3 & 4 & 5 \end{bmatrix}$$

into upper triangular form $E_{32}E_{31}E_{21} = U$? Multiply E_{32}^{-1}, E_{31}^{-1}, and E_{21}^{-1} to factor A into LU, where $L = E_{32}^{-1}E_{31}^{-1}E_{21}^{-1}$. Find L and U.

Problem 4: Use the Gauss-Jordan method to invert the matrices:

$$\begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} \quad \text{and} \quad \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix} \quad \text{and} \quad \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

Problem 5: Under what conditions is the following matrix invertible?

$$\begin{bmatrix} a & b & 0 \\ c & d & 0 \\ 0 & 0 & e \end{bmatrix}$$

Problem 6: True or false (give a counterexample if false and a reason if true):

(a) A 4×4 matrix with a row of zeros is not invertible.

(b) A matrix with 1s down the main diagonal is invertible.

(c) If A is an invertible matrix, then A^{-1} is invertible as well.

(d) If A^T is an invertible matrix, then A is invertible as well.

Problem 7: Suppose A is $m \times n$ matrix and B is a symmetric $m \times m$ matrix.

(a) Show that $A^T BA$ is symmetric; what size has this matrix?

(b) Show why $A^T A$ has no negative numbers on its diagonal.

Problem 8: Producing x_1 trucks and x_2 planes requires $x_1 + 50x_2$ tons of steel, $40x_1 + 1000x_2$ pounds of rubber, and $2x_1 + 50x_2$ months of labor. If the unit costs y_1, y_2, y_3 are \$700 per ton, \$3 per pound. and \$3000 per month, what are the values of one truck and one plane?