Ma 232 Linear Algebra **Darinka Dentcheva**
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Homework 3

(due Thursday, September 25, 2008)

Problem 1: Are the following sets vector spaces and why?

(a) the set of 2×2 nonsingular matrices;

(b) the set of all sequences $(a_1, a_2, \dots, a_n, \dots)$ satisfying the property

$$\sum_{i=1}^{\infty} |a_i| < \infty.$$

Problem 2: Find the largest possible number of independent vectors among

$$v^1 = \begin{bmatrix} 1 \\ -1 \\ 0 \\ 0 \end{bmatrix} \quad v^2 = \begin{bmatrix} 1 \\ 0 \\ -1 \\ 0 \end{bmatrix} \quad v^3 = \begin{bmatrix} 1 \\ 0 \\ 0 \\ -1 \end{bmatrix} \quad v^4 = \begin{bmatrix} 0 \\ 1 \\ -1 \\ 0 \end{bmatrix} \quad v^5 = \begin{bmatrix} 0 \\ 1 \\ 0 \\ -1 \end{bmatrix} \quad v^6 = \begin{bmatrix} 0 \\ 0 \\ 1 \\ -1 \end{bmatrix}$$

The number is the ..... of the space spanned by the vectors $v^1, v^2, \dots, v^6$.

Problem 3: Describe the subspace of $\mathbb{R}^3$ spanned by

(a) the two vectors $(1, 1, -1)$ and $(-1, -1, 1)$.

(b) the three vectors $(0, 1, 1)$, $(1, 1, 0)$, and $(0, 0, 0)$.

(c) all vectors with positive components.

What geometrical objects do these sets constitute?

Problem 4: Find a basis for each of the following subspaces of $\mathbb{R}^4$:

(a) All vectors whose components are equal.

(b) All vectors whose components add to 0.

(c) All vectors that are perpendicular to $(1, 1, 0, 0)$ and $(1, 0, 1, 1)$.

Problem 5: Suppose $v^1, v^2, \ldots, v^6$ are six vectors in $\mathbb{R}^4$. What is the correct statement?

(a) Those vectors (do)(do not)(might not) span $\mathbb{R}^4$.

(b) Those vectors (are)(are not)(might be) linearly independent.

(c) Any four of those vectors (are)(are not)(might not) a basis for $\mathbb{R}^4$.

Support your answer by a reason.

Problem 6: Find a basis for the space of bounded sequences l_∞.