Ma 232 Linear Algebra **Darinka Dentcheva**
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Homework 4

(due Thursday, October 2, 2008)

Problem 1: Use elimination procedure to find the rank of the following matrix:

$$\begin{bmatrix} 1 & 2 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 1 & 2 & 0 & 1 \end{bmatrix}$$

Which variables are free?

Problem 2: Write the complete solutions $x = x_p + x_n$ to the following system of equations:

$$\begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 5 \end{bmatrix} \begin{bmatrix} u \\ v \\ w \end{bmatrix} = \begin{bmatrix} 1 \\ 4 \end{bmatrix}.$$

Problem 3: Suppose A and B are $n \times n$ matrices and $AB = I_n$. Assume that $\text{rank}(AB) \leq \text{rank}(A)$. Prove that $\text{rank}(A) = n$. Is A invertible and is it true that $BA = I_n$ (provide reasons either way)?

Problem 4: Find the column space, the nullspace, and the solutions of the system $Ax = b$ for

$$A = \begin{bmatrix} 2 & 4 & 6 & 4 \\ 2 & 5 & 7 & 6 \\ 2 & 3 & 5 & 2 \end{bmatrix} \qquad b = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} = \begin{bmatrix} 4 \\ 3 \\ 5 \end{bmatrix}.$$

Problem 5: Construct a matrix whose column space contains the vectors

$$\begin{bmatrix} 1 \\ 1 \\ 3 \end{bmatrix} \quad \text{and} \quad \begin{bmatrix} 0 \\ 2 \\ 1 \end{bmatrix}$$

and whose nullspace contains

$$\begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix}.$$

Problem 6: Without elimination find the dimensions and basis for the four subspaces of

$$A = \begin{bmatrix} 0 & 3 & 3 & 3 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} 1 & 1 \\ 2 & 2 \\ 5 & 5 \end{bmatrix}$$

Problem 7: True or false (with a reason or a counterexample)?

(a) The matrices A and A^T have the same number of pivots.

(b) A and A^T have the same left nullspace.

(c) If the row space of A equals the column space of A, then $A^T = A$.