Ma 232 Linear Algebra **Darinka Dentcheva**
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Homework 9

(due Thursday, November 20, 2008)

Problem 1: Find all eigenvectors and eigenvalues of

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

and write two different diagonalization matrices S.

Problem 2: Suppose that the matrix A has eigenvalues 1,2,4. What is the trace of A^2? What is the determinant of $(A^{-1})^{\mathsf{T}}$?

Problem 3: Represent the matrix B given below in the form $S\Lambda S^{-1}$; then use this representation to prove the formula for B^k:

$$B = \begin{bmatrix} 3 & 1 \\ 0 & 2 \end{bmatrix} \quad B^k = \begin{bmatrix} 3^k & 3^k - 2^k \\ 0 & 2^k \end{bmatrix}.$$

Problem 4:

$$\text{If} \quad A = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix} \quad \text{and} \quad AB = BA, \quad \text{show that} \quad B = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

is also diagonal. The matrix B has the same eigen.............. as A, but different eigen...............

Problem 5: When $a + b = c + d$, show that $(1, 1)^T$ is an eigenvector and find both eigenvalues:

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

Problem 6: Choose a, b, c, so that $\det(A - \lambda I) = 9\lambda - \lambda^3$ and the eigenvalues of A are -3, 0, 3:

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ a & b & c \end{bmatrix}.$$