Ma 221 - Exam I review

First Order Differential Equations

Separable Equations

$$\frac{dy}{dx} = g(x)p(y)$$
$$h(y)dy = g(x)dx$$
$$\int h(y)dy = \int g(x)dx$$

Linear Equations

$$\frac{dy}{dx} + p(x)y = q(x)$$

Integrating Factor

$$IF = e^{\int p(x)dx}$$
$$e^{\int p(x)dx}\left(\frac{dy}{dx} + p(x)y\right) = e^{\int p(x)dx}[q(x)]$$
$$\frac{d}{dx}\left(e^{\int p(x)dx}y\right) = e^{\int p(x)dx}[q(x)]$$
$$e^{\int p(x)dx}y = \int\left(e^{\int p(x)dx}[q(x)]\right)dx$$
$$y = e^{-\int p(x)dx}\int\left(e^{\int p(x)dx}[q(x)]\right)dx$$

Exact Equations

$$M(x,y)dx + N(x,y)dy = 0$$

Test for exactness

$$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$$

When test is passed, there is $F(x,y)$ such that

$$M = \frac{\partial F}{\partial x} \quad and \quad N = \frac{\partial F}{\partial y}$$

Find $F(x,y)$. Solution is

$$F(x,y) = c$$

Bernoulli D.E.

$$\frac{dy}{dx} + p(x)y = q(x)y^n$$

$$y^{-n}\frac{dy}{dx} + p(x)y^{1-n} = q(x)$$

Substitution

$$z = y^{1-n}$$

$$\frac{dz}{dx} = (1-n)y^{-n}\frac{dy}{dx}$$

yields a linear d.e. in z.

In all cases, the arbitrary constant resulting from integration is used to satisfy any initial condition.