Ma 221 Homework Solutions Due Date: January 29, 2015

2.2 pg. 43 # 2, 3, 6, <u>11</u>, <u>15</u>, 17 <u>19</u>, 21, <u>23</u>, 25; (Underlined problems are handed in)

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In problems 1, 4 and 5, determine whether the given differential equation is separable.

2) $\frac{dy}{dx} = 4y^2 - 3y + 1 \Rightarrow \quad dy = (4y^2 - 3y + 1)dx \Rightarrow \quad \frac{dy}{4y^2-3y+1} = dx$

therefore, this equation is separable.

3.) $\frac{ds}{dt} = t\ln(s^{2t}) + 8t^2 \Rightarrow \quad \frac{ds}{dt} = 2t^2 \ln s + 8t^2 \Rightarrow \quad \frac{ds}{dt} = 2t^2(\ln s + 4) \Rightarrow$

$$\frac{ds}{\ln s + 4} = 2t^2 dt$$

therefore, this equation is separable.

6.) $s^2 + \frac{ds}{dt} = \frac{s+1}{st}$

Writing the equation in the form

$$\frac{ds}{dt} = \frac{s+1}{st} - s^2$$

shows that the equation is not separable.

In problems 11 and 15, solve the equation.

11) $\frac{dy}{dx} = \frac{\sec^2 y}{1+x^2}$

$\frac{dy}{dx} = \frac{\sec^2 y}{1+x^2} \Rightarrow \quad \frac{dy}{\sec^2 y} = \frac{dx}{1+x^2}$

Using trigonemtric identities we have:

$\Rightarrow \sec y = \frac{1}{\cos y}$ and $\cos^2 y = \frac{1}{2}(1 + \cos 2y)$

$\Rightarrow \frac{dy}{\sec^2 y} = \frac{dx}{(1+x^2)} \Rightarrow \quad \frac{(1+\cos 2y)dy}{2} = \frac{dx}{1+x^2} \Rightarrow \quad \int \frac{(1+\cos 2y)dy}{2} = \int \frac{dx}{1+x^2}$

$\Rightarrow \quad \frac{1}{2}(y + \frac{1}{2}\sin 2y) = \arctan x + C_1 \Rightarrow \quad 2y + \sin 2y = 4\arctan x + 4C_1$

$\Rightarrow \quad 2y + \sin 2y = 4\arctan x + C$

15.) $y^{-1}dy + ye^{\cos x}\sin x dx = 0$

$$-ye^{\cos x}\sin x dx = y^{-1}dy$$
$$-\int e^{\cos x}\sin x dx = \int y^{-2}dy$$

Substituting:

$$\text{let } u = \cos x$$
$$du = -\sin x dx$$
$$-du = \sin x dx$$

$$\int e^u du = \int y^{-2}dy$$
$$e^u = -y^{-1} + C$$
$$e^{\cos x} = y^{-1} + C$$
$$y^{-1} = \frac{1}{C - e^{\cos x}}$$

In problems 19, 21, 23, and 25, solve the initial value problem.

19) $\frac{dy}{dx} = 2\sqrt{y+1}\cos x, \qquad y(\pi) = 0$

$\frac{dy}{2\sqrt{y+1}} = \cos x dx = \frac{1}{2}(y+1)^{-1/2}dy = \cos x dx \Rightarrow \qquad \int \frac{1}{2}(y+1)^{-1/2}dy = \int \cos x dx$

$(y+1)^{1/2} = \sin x + C$ Substituting the IC $y(\pi) = 0 \Rightarrow$

$(0+1)^{1/2} = \sin\pi + C \Rightarrow \qquad C = 1$

$(y+1)^{1/2} = \sin x + 1 \Rightarrow \qquad y = (\sin x + 1)^2 - 1 = \sin^2 x + 2\sin x$

21.)

$$\frac{1}{\theta}\frac{dy}{d\theta} = \frac{y\sin\theta}{y^2+1}, \qquad y(\pi) = 1$$

$$\frac{y^2+1}{y}dy = \theta\sin\theta d\theta$$
$$ydy + \frac{1}{y}dy = \theta\sin\theta d\theta$$

Integrating

$$\frac{y^2}{2} + \ln y = \sin\theta - \theta\cos\theta + C$$

The IC implies

$$\frac{1}{2} = \pi + C$$

so

$$y^2 + \ln y = \sin\theta - \theta\cos\theta + \frac{1}{2} - \pi$$

23.) $t^{-1}\frac{dy}{dt} = 2\cos^2 y\,,\quad y(0) = \frac{\pi}{4}$

$$\frac{dy}{\cos^2 y} = 2tdt$$

$$\int \sec^2 y dy = \int 2tdt$$

$$\tan y = t^2 + C$$

$$\tan(\frac{\pi}{4}) = C$$

$$C = 1$$

Thus

$$\tan y = t^2 + 1$$

or

$$y = \arctan(t^2 + 1)$$

25.) $\frac{dy}{dx} = x^2(1+y) \qquad y(0) = 3$

$\frac{dy}{dx} = x^2(1+y) \Rightarrow \qquad \int \frac{dy}{1+y} = \int x^2 dx \Rightarrow \qquad \ln(1+y) = \frac{1}{3}x^3 + C_1 \Rightarrow$

$$1 + y = e^{\frac{1}{3}x^3 + C_1} = Ce^{\frac{1}{3}x^3}$$

$\Rightarrow \qquad y = Ce^{\frac{1}{3}x^3} - 1$

$3 = y(0) = C - 1 \Rightarrow \qquad C = 4 \Rightarrow$

$$y = 4e^{\frac{1}{3}x^3} - 1$$