MA 221 Homework Solutions
Due February 19, 2015

4.2 p. 166 # 37, 43
4.4 p. 182 # 10, 11, 14
(Underlined problems are to be handed in)

Section 4.2

37) For problem 37, find three linearly independent solutions.

$$y''' + y'' - 6y' + 4y = 0$$

The auxiliary equation is

$$r^3 + r^2 - 6r + 4 = 0$$

$r = 1$ is clearly a root. So $r^3 + r^2 - 6r + 4 = (r-1)(r^2 + 2r - 4) = 0$
Thus

$$r = 1 \text{ and } r = \frac{-2 \pm \sqrt{4 - 4(1)(-4)}}{2} = -1 \pm \sqrt{5}$$

Hence

$$y(t) = c_1 e^t + c_2 e^{(-1+\sqrt{5})t} + c_3 e^{(-1-\sqrt{5})t}$$

For problem 43, solve the initial value problem.
43.)
$y''' - y' = 0, y(0) = 2, y'(0) = 3, y''(0) = -1$
$r^3 - r = 0$
$r = 1, -1, 0$
$y(t) = c_1 e^{0t} + c_2 e^{-t} + c_3 e^t = c_1 + c_2 e^{-t} + c_3 e^t$
$y'(t) = -c_2 e^{-t} + c_3 e^t$
$y''(t) = c_2 e^t + c_3 e^t$
$y(0) = c_1 + c_2 + c_3 = 2$
$y'(0) = -c_2 + c_3 = 3$
$y''(0) = c_2 + c_3 = -1$
$c_1 = 3$
$c_2 = -2$
$c_3 = 1$
$\Rightarrow y(t) = 3 - 2e^{-t} + e^t$

Section 4.4
Find a particular solution to the differential equation.
10)

$$y'' + 3y = -9$$

By inspection we see that $y_p = -3$.

11)

$$y''(x) + y(x) = 2^x$$

Let $y_p = A_0 2^x \Rightarrow y_p' = A_0(\ln 2)2^x$ and $y_p'' = A_0(\ln 2)^2 2^x$. The DE implies

$$y_p'' + y_p = A_0(\ln 2)^2 2^x + A_0 2^x = A_0\left[(\ln 2)^2 + 1\right]2^x = 2^x$$

Hence

$$A_0 = \frac{1}{(\ln 2)^2 + 1}$$

and

$$y_p = \frac{2^x}{(\ln 2)^2 + 1}$$

14)

$$2z'' + z = 9e^{2t}$$

The auxiliary equation for the homogeneous equation is $p(r) = 2r^2 + 1 = 0$. Thus e^{2t} is not a homogeneous solution. Hence the formula

$$z_p = \frac{Ae^{\alpha t}}{p(\alpha)}$$

and $A = 9, \alpha = 2$ so

$$z_p = \frac{9e^{2t}}{9} = e^{2t}$$