Ma 221 Homework Solutions Due 2/26/15

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18.

$$y'' + 4y = 8\sin 2t$$

The characteristic polynomial is $p(r) = r^2 + 4$ so the roots are $r = \pm 2i$ and therefore

$$y_h = c_1 \sin 2t + c_2 \cos 2t$$

Consider a companion equation

$$v'' + 4v = 8\cos 2t$$

Multiplying the first equation by i and adding it to the second equation yields

$$v'' + iy'' + v + iy = 8(\cos 2t + i\sin 2t) = 8e^{2it}$$

Let $w = v + iy.$ Then we have

$$w'' + 4w = 8e^{2it}$$

Since $p(2i) = 0,$ but $p'(2i) = 4i \neq 0,$ then

$$w_p = \frac{8te^{2it}}{4i} = 2\frac{te^{2it}}{i}$$

$y_p = \operatorname{Im} w_p$

$$w_p = 2\frac{te^{2it}}{i} \times \frac{i}{i} = -2ite^{2it}$$
$$= -2it(\cos 2t + i\sin 2t)$$

Thus

$$y_p = -2t\cos 2t$$

19.

$$4y'' + 11y' - 3y = -2te^{-3t}$$

Consider the homogeneous equation $4y'' + 11y' - 3y = 0$ first.

$$p(r) = 4r^2 + 11r - 3 = 0$$

$$r = \frac{-11 \pm \sqrt{121 - 4(4)(-3)}}{2(8)} = \frac{-11 \pm \sqrt{169}}{2(8)} = \frac{-11 \pm 13}{16} = -3, \frac{1}{8}$$

Thus e^{-3t} is a homogeneous solution.

$$y_p = t(A_1 t + A_0)e^{-3t} = \left(A_1 t^2 + A_0 t\right)e^{-3t}$$

$$y_p' = \left[-3A_1 t^2 + (2A_1 - 3A_0)t + A_0\right]e^{-3t}$$

$$y_p'' = \left[9A_1 t^2 + (9A_0 - 12A_1)t + 2A_1 - 6A_0\right]e^{-3t}$$

Substituting into the DE yields after some algebra

$$[-26A_1 t + (8A_1 - 13A_0)]e^{-3t} = -2te^{-3t}$$

Thus

$$\begin{aligned} -26A_1 &= -2 \\ 8A_1 - 13A_0 &= 0 \end{aligned}$$

Hence $A_0 = \frac{8}{169}, A_1 = \frac{1}{13}$ and

$$y_p = \left(\frac{t}{13} + \frac{8}{169}\right)te^{-3t}$$

24.

$$y'' + y = 4x\cos x$$

Since $\cos x$ and $\sin x$ are homogeneous solutions, the we let

$$y_p = \left(A_1x^2 + A_0x\right)\cos x + \left(B_1x^2 + B_0x\right)\sin x$$

Thus

$$y_p' = \left[B_1x^2 + (B_0 + 2A_1)x + A_0\right]\cos x + \left[-A_1x^2 + (2B_1 - A_0)x + B_0\right]\sin x$$

and

$$y_p'' = \left[-A_1x^2 + (4B_1 - A_0)x + 2(B_0 + A_1)\right]\cos x + \left[-B_1x^2 + (-4A_1 - B_0)x + 2(B_1 - A_0)\right]\sin x$$

Substituting into the DE and combining yields

$$[4B_1x + 2(B_0 + A_1)]\cos x + [-4A_1x + 2(B_1 - A_0)]\sin x = 4x\cos x$$

Hence

$$\begin{aligned} 4B_1 &= 4 \quad \text{or} \quad B_1 = 1 \\ -4A_1 &= 0 \quad \text{so} \quad A_1 = 0 \\ 2(B_0 + A_1) &= 0 \quad \text{so} \quad B_0 = -A_1 = 0 \\ 2(B_1 - A_0) &= 0 \quad \text{so} \quad A_0 = B_1 = 1 \end{aligned}$$

Thus

$$y_p = x\cos x + x^2\sin x$$

34.

$$2y''' + 3y'' + y' - 4y = e^{-t}$$

The characteristic equation is

$$p(r) = 2r^3 + 3r^2 + r - 4 = 0$$

$p(-1) = -2 + 3 - 1 - 4 = -4 \neq 0.$ Thus e^{-t} is not a homogeneous solution and

$$y_p = Ae^{-t}$$

$y_p' = -Ae^{-t}$, $y_p'' = +Ae^{-t}$, $y_p''' = -Ae^{-t}$ Substituting into the DE we have

$$(-2A + 3A - A - 4A)e^{-t} = e^{-t}$$

or $-4A = 1$ so

$$y_p = -\frac{1}{4}e^{-t}$$