Homework 1 Solutions

Problem 2.6

Two jurors are selected from 4 alternates to serve at a murder trial. Using the notation A_1A_3, for example, to denote the simple event that alternates 1 and 3 are selected, list the 6 elements of the sample space.

Solution: $S = \{A_1A_2, A_1A_3, A_1A_4, A_2A_3, A_2A_4, A_3A_4\}$.
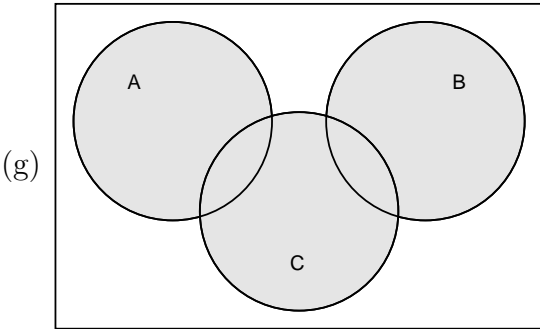
Problem 2.8

An experiment involves tossing a pair of dice, 1 green and 1 red, and recording the numbers that come up. If x equals the outcome on the green die and y the outcome on the red die, let (x, y) denote the corresponding element of the sample space S. For this sample space:

(a) list the elements corresponding to the event A that the sum is greater than 8;

(b) list the elements corresponding to the event B that a 2 occurs on either die;

(c) list the elements corresponding to the event C that a number greater than 4 comes up on the green die;

(d) list the elements corresponding to the event $A \cap C$;

(e) list the elements corresponding to the event $A \cap B$;

(f) list the elements corresponding to the event $B \cap C$;

(g) construct a Venn diagram to illustrate the intersections and unions of the events A, B, and C.

Solution:

(a) $A = \{(3, 6), (4, 5), (4, 6), (5, 4), (5, 5), (5, 6), (6, 3), (6, 4), (6, 5), (6, 6)\}$.

(b) $B = \{(1, 2), (2, 2), (3, 2), (4, 2), (5, 2), (6, 2), (2, 1), (2, 3), (2, 4), (2, 5), (2, 6)\}$.

(c) $C = \{(5, 1), (5, 2), (5, 3), (5, 4), (5, 5), (5, 6), (6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6)\}$.

(d) $A \cap C = \{(5, 4), (5, 5), (5, 6), (6, 3), (6, 4), (6, 5), (6, 6)\}$.

(e) $A \cap B = \emptyset$.

(f) $B \cap C = \{(5, 2), (6, 2)\}$.

(g)

Problem 2.10

An engineering firm is hired to determine if certain waterways in Virginia are safe for fishing. Samples are taken from three rivers.

(a) List the elements of a sample space S, using the letters F for "safe to fish" and N for "not safe to fish."

(b) List the elements of S corresponding to event E that at least two of the rivers are safe for fishing.

(c) Define an event that has as its elements the points

$$\{FFF, NFF, FFN, NFN\}.$$

Solution:

(a) $S = \{FFF, FFN, FNF, NFF, FNN, NFN, NNF, NNN\}$.

(b) $E = \{FFF, FFN, FNF, NFF\}$.

(c) The second river is safe for fishing.

Problem 2.14

If $S = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$ and $A = \{0, 2, 4, 6, 8\}$, $B = \{1, 3, 5, 7, 9\}$, $C = \{2, 3, 4, 5\}$, and $D = \{1, 6, 7\}$, list the elements of the sets corresponding to the following events:

(a) $A \cup C$;

(b) $A \cap B$;

(c) C';

(d) $(C' \cap D) \cup B$;

(e) $(S \cap C)'$;

(f) $A \cap C \cap D'$.

Solution:

(a) $A \cup C = \{0, 2, 3, 4, 5, 6, 8\}$.

(b) $A \cap B = \emptyset$.

(c) $C' = \{0, 1, 6, 7, 8, 9\}$.

(d) $C' \cap D = \{1, 6, 7\}$, so $(C' \cap D) \cup B = \{1, 3, 5, 6, 7, 9\}$.

(e) $(S \cap C)' = C' = \{0, 1, 6, 7, 8, 9\}$.

(f) $A \cap C = \{2, 4\}$, so $A \cap C \cap D' = \{2, 4\}$.

Problem 2.16

If $S = \{x | 0 < x < 12\}$, $M = \{x | 1 < x < 9\}$, and $N = \{x | 0 < x < 5\}$, find
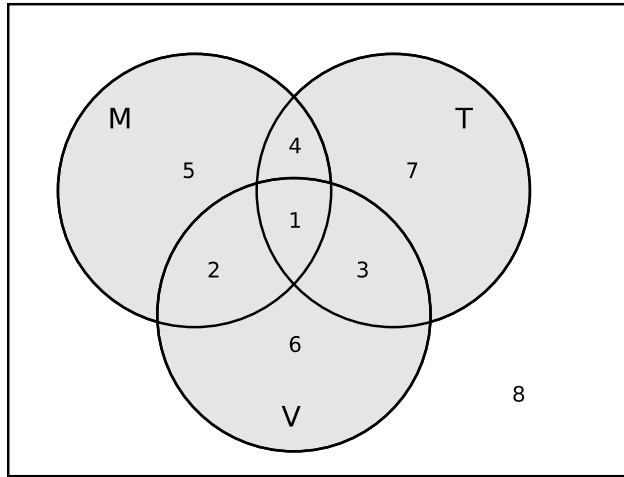
(a) $M \cup N$;

(b) $M \cap N$;

(c) $M' \cap N'$.

Solution:

(a) $M \cup N = \{x \mid 0 < x < 9\}$.

(b) $M \cap N = \{x \mid 1 < x < 5\}$.

(c) $M' \cap N' = \{x \mid 9 \leq x < 12\}$.

Problem 2.20

Suppose that a family is leaving on a summer vacation in their camper and that M is the event that they will experience mechanical problems, T is the event that they will receive a ticket for committing a traffic violation, and V is the event that they will arrive at a campsite with no vacancies. Referring to the Venn diagram below, list the numbers of the regions that represent the following events:

(a) The family will experience no mechanical problems and commit no traffic violation but will arrive at a campsite with no vacancies.

(b) The family will experience both mechanical problems and trouble in locating a campsite with a vacancy but will not receive a ticket for a traffic violation.

(c) The family will either have mechanical trouble or arrive at a campsite with no vacancies but will not receive a ticket for committing a traffic violation.

(d) The family will not arrive at a campsite with no vacancies.

Solution:

(a) 6;

(b) 2;

(c) 2, 5, 6;

(d) 4, 5, 7, 8.