1. [20 pts] Find the equation of the plane, which passes through points \( A(7, 2, -3) \) and \( B(5, 6, -4) \) parallel to the \( x \)-axis.

**Solution.** We see that vectors \( \vec{i} = \langle 1, 0, 0 \rangle \) and \( \vec{AB} = \langle -2, 4, -1 \rangle \) are parallel to the plane. Then normal vector \( \vec{n} = \vec{i} \times \vec{AB} \), and we obtain

\[
\vec{n} = \begin{vmatrix}
\vec{i} & \vec{j} & \vec{k} \\
1 & 0 & 0 \\
-2 & 4 & -1
\end{vmatrix} = \langle 0, 1, 4 \rangle.
\]

Answer: \( y - 2 + 4(z + 3) = 0 \) or \( y + 4z = -10 \).

2. [20 pts] Determine whether the lines are parallel, skew or intersecting. If they intersect, find the points of intersection:

\[
x - 1 \quad y - 7 \quad z - 3 \\
2 \quad 4 \quad 4 \\
3 \quad -2 \quad 1
\]

**Solution.** In parametric form we obtain

\[
\begin{cases}
x = 1 + 2t \\
y = 7 + t \\
z = 3 + 4t
\end{cases}, \quad \begin{cases}
x = 6 + 3s \\
y = -1 - 2s \\
z = -2 + s
\end{cases}
\]

Equalling these equations, we obtain

\[
\begin{cases}
1 + 2t = 6 + 3s \\
7 + t = -1 - 2s \\
3 + 4t = -2 + s
\end{cases}
\]

Then the solution to the above system \( t = -2, s = -3 \) provides us the intersection point \( P(-3, 5, -5) \).
3. [20 pts] Find the length of the curve

\[ \vec{r}(t) = \langle 8 \cos \frac{t}{2}, \frac{15}{t}, 4 \sin t \rangle, \quad 0 \leq t \leq \frac{\pi}{2}. \]

**Solution.** \( L = \int_a^b |\vec{r}'(t)| dt \). In our case \( \vec{r}'(t) = (-4 \sin \frac{t}{2}, 0, 4 \cos t) \). Then

\[ L = \int_0^{\pi/2} \sqrt{16 \cos^2 \frac{t}{2} + 16 \cos^2 t} \, dt = 4 \int_0^{\pi/2} \sqrt{\cos^2 \frac{t}{2} + \cos^2 t} \, dt. \]

4. [20 pts] Find the distance between point \( A(1, 2, -3) \) and the plane \( 4x - 3z - 1 = 0 \).

**Solution.** To find any point from the plane, let \( z = 1 \) and then \( x = 1 \). So, point \( P(1, 0, 1) \in \text{plane} \). The distance is equal to the magnitude of the projection of vector \( \vec{AP} \) onto the normal direction to the plane. Since \( \vec{AP} = (0, -2, 4), \vec{n} = (4, 0, -3), \)

\[ d = |\text{comp}_\vec{n} \vec{AP}| = \left| \frac{\vec{AP} \cdot \vec{n}}{||\vec{n}||} \right| = \frac{12}{5}. \]

5. [20 pts] Let \( \vec{r}(t) \) be the position vector of a particle. Write formula for its velocity \( \vec{v}(t) \) and acceleration \( \vec{a}(t) \).

**Solution.** \( \vec{v} = \vec{r}' \) and \( \vec{a} = \vec{r}'' \).