

**Y.A.Antipov** (Louisiana State Univ.)

**“Second-order difference equations with periodic coefficients in diffraction theory”**

An analytical method for a scalar second-order difference equation

$$a(s)f(s+h)+b(s)f(s)+c(s)f(s-h)=d(s), s \text{ in } \mathbf{C},$$

with meromorphic periodic coefficients is proposed. The technique involves reformulating the equation as a vector difference equation of the first order and reducing it to a scalar Riemann-Hilbert problem for two finite segments on a hyperelliptic surface. The final step of the procedure is the solution of the classical Jacobi's inversion problem. The method is illustrated by solving in closed form a second-order functional-difference equation when the corresponding surface is a torus.

The solution is constructed in terms of elliptic functions. Generalizations for (i) the case of meromorphic not necessarily periodic coefficients and (ii) difference equations of  $n$ -th order are discussed.

This technique is applied for the solution of the problem on electromagnetic ( $E$ -polarization) scattering by a right-angled magnetically conductive wedge. The physical problem reduces to a second-order difference equation with  $2\pi$ -periodic coefficients and with the shift  $\pi$ . A rigorous procedure for constructing the general solution is proposed. It consists of two steps. First, an auxiliary equation with the shift  $2\pi$  and the period  $\pi$  is derived and solved by the method of the Riemann-Hilbert problem on a torus. Next, necessary and sufficient conditions for the solution of the auxiliary equation to satisfy the governing equation are derived. These conditions separate the general solution of the main equation from those solutions of the auxiliary equation which fail to satisfy the governing difference equation. In addition, the particular case of no branch points is analyzed by the machinery of the Riemann-Hilbert problem for a segment on the complex plane. A high-frequency asymptotic expression for the electric field is presented. Numerical results for the backscattering coefficient are reported.

**Guillaume Bal** (Columbia Univ)

**“Kinetic and diffusive models for waves in random media”**

**Pavel Dubovski** (Stevens Inst)

**“A new model of the coagulation kinetics”**

The celebrated Smoluchowski coagulation equation assumes that two colliding particles just merge together with some probability. However, the experiments demonstrate that the increase in the particle size is overlapped by splitting the particles during a shorter time interval. We simulate such collisions and derive a new balance kinetic model. We compare analytically and numerically the results for both coagulation models (Smoluchowski and new one) and show a surprising coincidence in the results: both models have same or almost same time-dependent solutions, equilibria, and critical phase transition times.

**Li Guo** (Rutgers Univ., Newark)

**“Algebraic methods in renormalization of perturbative QFT”**

Algebraic structures have emerged in recently years that underlie the renormalization process of perturbative quantum field theory. In the work of Connes and Kreimer, Feynman graphs are organized into a Hopf algebra and a regularized Feynman rule is given by an algebra homomorphism from this Hopf algebra to a Rota-Baxter algebra. We will give an introduction of their work and explain how pQFT results such as the algebraic Birkhoff decomposition for renormalization and the Bogoliubov formula can be derived from theorems on Rota-Baxter algebras. We further show that the Feynman rules have a matrix representation that converts the process of renormalization to matrix calculations.

**Sandra Hayes** (Technische Universitat Munchen, Germany/Queens College CUNY)

**”Chaotic dynamical systems”**

**Michael Kiessling** (Rutgers University, New Brunswick)

**“Microscopic foundations of Vlasov theory”**

I review the current state of affairs in the microscopic foundations of Vlasov theory, beginning with the Newtonian works of Neunzert and Braun and Hepp and ending with some recent semi-relativistic joint work of myself and Y. Elskens and V. Ricci

**Gabriel Koch** (Univ. of Minnesota)

**LIOUVILLE THEOREM FOR 2-D NAVIER-STOKES EQUATIONS**

(Joint work with V. Sverak and N. Nadirashvili.)

ABSTRACT:

We prove a Liouville theorem for the Navier-Stokes equations in two space dimensions, that global bounded solutions  $\mathbf{u}(x, t) = (u_1(x, t), u_2(x, t))$ ,  $x \in \mathbb{R}^2$ ,  $t \in \mathbb{R}$ , of the equations

$$\begin{aligned}\mathbf{u}_t - \Delta \mathbf{u} + (\mathbf{u} \cdot \nabla) \mathbf{u} + \nabla p &= 0 \\ \operatorname{div} \mathbf{u} &= 0\end{aligned}$$

are of the form  $\mathbf{u}(x, t) = \mathbf{u}_0(t)$ . In particular, in the steady-state case, bounded solutions are constant. (Note that we do not need any assumptions regarding the pressure,  $p(x, t)$ .)

In the presentation of this work, I will outline both the relevant background as well as some related open problems for those unfamiliar with the topic.

**Chiu-Ya Lan** (National Sun Yet-sen Univ., Taiwan)

**“The Green's function for the 1-dimensional Broadwell Model”**

We consider the Broadwell model in  $\mathbb{R}^1$ . In this talk, we shall derive pointwise estimates for the Green's function of the linearized Broadwell model. Through these estimates, we will reveal clearly the behavior of the particle-like waves propagating with microscopic velocity, and the fluid-like waves as seen by the Chapman-Enskog expansion. These estimates can be used to study the time asymptotic behavior of the solutions to the initial-boundary value problem

**Marco Lenci** (Stevens Institute)

**"Classical delocalization vs. quantum localization for non-compact cusped billiards"**

We show examples of billiard systems on non-compact tables whose classical and quantum dynamics behave, at least for some aspects, in radically different ways. In particular, classical ergodicity entails that the material point delocalizes for long times, but the quantum eigenstates are (and remain at any time) strongly localized; and this while the Schnirelman Theorem--often referred to as quantum ergodicity--holds. We use these examples to discuss the scope of the definition of quantum ergodicity named after Schnirelman.

**Yi Li** (Stevens Inst)

**“Nonlinear dispersive approximation to the water wave problem”**

We investigate the nonlinear dispersive effect of the Green-Naghdi (GN) equations as second order approximations to the full water wave problem. Using the Hamiltonian structure of the GN system and comparison methods, we show that the GN system possesses some solutions that remain in a neighborhood of certain bounded, oscillating functions. This fact demonstrates the the nonlinear dispersion effect on the existence of global solutions to the GN system as a contrast to the dispersionless, first order shallow water approximations.

**Jacek Polewczak** (Univ. of California-Northridge)

**“Recent results in inert and reacting kinetic theories of dense fluids”**

**Joel Rogers** (Polytechnic University, Brooklyn)

**“Formulation of Some Free Boundary Problems in Multicomponent Mixtures as Conservation Laws with Constraints”**

The flow of an incompressible fluid with a free boundary has been reformulated as a system of conservation laws with a one-sided density constraint. An algorithm to solve these conservation laws with the constraint, motivated by kinetic theory, with a time step  $\Delta t$  as a parameter, has been developed to solve problems to which they apply. Specific applications are to flows with breaking waves, to sprays, and to underwater explosions. When there is a second incompressible fluid present, one has a more complicated set of conservation laws. In particular, we consider the case in which the second fluid is sand, modeled as a Bingham plastic. Now there are one-sided constraints on the volume

fraction of the sand as well as on the volume occupied by the combination of water and sand. The structure of these equations and implementation of an algorithm to solve them are indicated. The combined system admits, in addition to the applications listed above, the treatment of flows in the presence of saturated sand, unsaturated sand, and slurries.

**Schrodinger operators with singular potentials: Lieb-Thirring bounds  
and preservation of a.c. spectrum**

ALEXEI RYBKIN  
Univ of Alaska Fairbanks, USA

We introduce a new series of trace type relations for 1D Schrodinger operators generalizing the well-known Faddeev-Zhakharov trace formulas. Comparing to the original Faddeev-Zhakharov formulas, these relations look quite unwieldy but have an important advantage - they hold for large classes of distributional (singular) potentials. We intend to introduce new Lieb-Thirring type bounds and discuss the absolutely continuous spectrum preservation in the context of the full line Schrodinger operator with potentials from some Sobolev spaces with negative indexes.

**Becca Thomases** (Courant Inst of Math Sciences)

**"Local Energy Decay for Solutions of Isotropic Symmetric Hyperbolic Systems and Applications to Nonlinear Elasticity"**

A unifying approach for obtaining weighted  $L^2$  estimates for solutions to a class of multi-dimensional symmetric hyperbolic systems with constraints is presented. The main assumptions are that the system is isotropic and possesses a useful set of commuting vector fields. This type of estimate is an important step in obtaining dispersive estimates necessary for the construction of solutions to nonlinear problems. Global solutions to the equations of motion for incompressible nonlinear elasticity are constructed using these techniques.

**Pangyen Weng** (Ramapo College, NJ)

**"On the Identification of Two Sobolev Spaces in Fluid Mechanics"**

This talk is on two Sobolev spaces of divergence-free vector fields, the equivalence of which is still an open problem. We first introduce briefly the importance of this problem and some of the existing partial results. We then present a new result which establishes the equivalence of the two spaces in 2-dimension. We will sketch a proof based on a theorem by Hedberg. If time permits, we will then provide some examples of immediate applications.

**Takayuki Yamauchi** (Lincoln Univ., PA)

**"On the Regularity of a Minimizer of an Energy Functional over a Class of Continuous Rearrangements"**

A state of equilibrium of a plasma flow in a nuclear fusion reactor in ideal magnetohydrodynamics (MHD) is achieved as a minimizer of the magnetic flux energy over a class of continuous rearrangements of a prescribed smooth profile magnetic flux function in  $H_0^1(\Omega)$ , where  $\Omega$  is the cross section of the nuclear fusion reactor. It was shown in 1985 that an energy minimizer is real analytic when  $\Omega$  is a disk. It has been conjectured in 1985 that an energy minimizer should be smooth when the boundary of  $\Omega$  is smooth. A sufficient condition for the existence of a smooth minimizer for a general open and bounded  $\Omega$  having a smooth boundary is presented in this work.

**Michael Zabaranin** (Stevens Inst)

**“Generalized analytic functions in hydrodynamics of axially symmetric Stokes flows”**

Hilbert formulas for an  $r$ -analytic function, defined by a generalized Cauchy--Riemann system in the domain exterior to the contours of a spindle and a lens in the meridional cross-section plane, have been derived. The derivation is based on the theory of Riemann boundary-value problems for analytic functions. For numerical calculations, Fourier and Mehler-Fock integrals with Hilbert formulas representing the real and imaginary parts of the  $r$ -analytic function have been reduced to the form of regular integrals. The problems of the axially symmetric steady motion of a rigid spindle-shaped and lens-shaped bodies in a Stokes fluid has been solved, and the pressure in the fluid has been expressed analytically based on a Hilbert formula. As an illustration, streamlines about the bodies, vortex and pressure functions at the contour of the bodies, and the drag force, exerted on the bodies by the fluid, have been calculated.

**Maxim Zyskin** (Univ. of Bristol, UK)

**“Nematic Liquid Crystals in Polyhedral Domains”**

Our study of nematic liquid crystals in polyhedral domains is motivated by applications to bi-stable displays (as well as interesting mathematics). Configurations of nematic liquid crystal are described by director fields. Boundary conditions, determined by surface treatment, are often tangent boundary conditions: on faces of polyhedron, director field must be tangent to the face. Such configurations are necessarily discontinuous at vertices. Assuming that director field is continuous away from vertices, there are many topologically inequivalent configurations of nematic. These topologically inequivalent configurations are classified by topological invariants (edge signs, kink numbers, wrapping numbers). Stable configurations of nematic liquid crystal are local minima of certain energy functional, the Frank energy. In the so-called one-constant approximation, Frank energy coincides with the Dirichlet energy. We establish lower energy bounds for minimal Dirichlet energy for configurations of a given topological type. We also establish upper energy bounds in the case when domain is a rectangular prism. Analytic and numerical evidence indicates that in a rectangular prism, there are minimising configurations of simplest topology which are regular away from the vertices, while for nontrivial topologies it is often energetically preferable for topologically nontrivial behaviour to occur in thin tubes along the edges. For some topological types, there is a transition from regular to singular-on-edge behaviour, depending on the aspect ratios of the prism.