SEMI-GLOBAL OUTPUT FEEDBACK STABILIZATION FOR THE NONLINEAR BENCHMARK EXAMPLE

Z. P. Jiang ,   D. J. Hill ,   Y. Guo

Department of Electrical Engineering
The University of Sydney
NSW 2006
AUSTRALIA
fax: +61 2 9351 5132 and e-mail: {zjiang, davidh, guo}@ee.usyd.edu.au

Keywords : Semiglobal stabilization; nonlinear control; output feedback; observer.

Abstract

In this paper we prove that the nonlinear benchmark RTAC example is semi-globally stabilizable using output feedback when the angle of the proof mass and/or the translational position of the cart are considered as the output. The proposed result complements previous work which requires exact knowledge of the translational and/or angular velocities but achieves global stabilization.

1 Introduction

The Rotational/Translational ACtuator (RTAC) system, first introduced in [12], serves as a nonlinear benchmark example in a number of recent papers [12, 2, 6, 7, 11] for purposes of comparing and testing different control methodologies. As far as global stabilization is concerned, two main ideas have been utilized: integrator backstepping and passivity. The advantages and drawbacks of each approach have been discussed in detail in [6]. Unfortunately, for this benchmark RTAC example, all the globally asymptotically stabilizing controllers proposed so far require the information of full-state or partial-state including velocities of the nonlinear RTAC system. Since the submission of the original manuscript, we became aware of the work [3] by Ortega where a GAS control law without velocity measurements was proposed on the basis of energy shaping and damping injection.

Motivated by the fact that requiring the precise measurement of translational and angular velocities of a mechanical system is often difficult or costs too much in practice, we consider in this paper the problem of output feedback stabilization. For the RTAC system, the angle of the proof mass and/or the translational displacement of the cart are regarded as the output. We do this at the price of achieving only semi-global stabilization, that is, given any compact set S where the initial conditions depart, there exists an output feedback controller such that the equilibrium of interest is asymptotically stable with a basin of attraction containing the compact set S. The interested reader is referred to [1, 8, 10] for motivation and references for semi-global stabilization.

2 RTAC Model and Problem Statement

The picture drawn in Figure 1 represents the benchmark Rotational/Translational ACtuator (RTAC) system which is composed of a translational oscillator connected with a linear spring of stiffness k fixed at a wall and having an eccentric rotational proof.

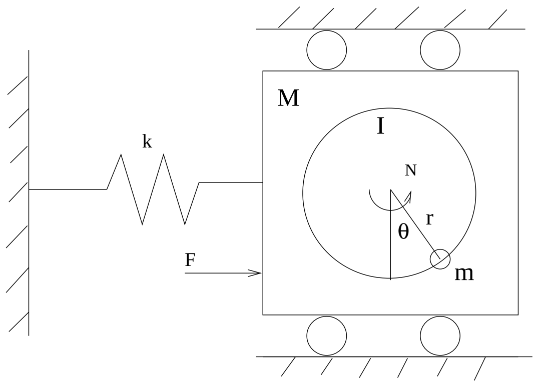

Figure 1: Nonlinear benchmark example [12]

Assume the cart moves only in the horizontal direction. Because the rotor axle is vertical, there is no gravitation effect. The motion of the oscillations is described by:

$$(M+m)\ddot{x}_c + mr(\ddot{\theta}\cos\theta - \dot{\theta}^2\sin\theta) + kx_c = F \quad (1)$$

$$(I+mr^2)\ddot{\theta} + m\ddot{x}_c r\cos\theta = N \quad (2)$$

where F is the disturbance force acting on the cart, N is the control torque applied to the proof mass, x_c is the translational position and θ is the rotational angle. In (1) and (2), M represents the total mass of the disk and the cart, m and I represent the mass and the moment of inertia of the eccentric mass, respectively, and r is the radius of rotation.

After suitable normalized transformation in the space of state variables, control input and time, the equations (1) and (2) are rewritten as (see [12]):

$$\begin{aligned} \ddot{x}_d + x_d &= \varepsilon(\dot{\theta}^2 \sin\theta - \ddot{\theta}\cos\theta) + F_d && (3) \\ \ddot{\theta} &= u - \varepsilon \ddot{x}_d \cos\theta && (4) \end{aligned}$$

where $0 < \varepsilon < 1$ and u is the control input.

Introducing the change of coordinates:

$$\begin{aligned} x_1 &= x_d + \varepsilon \sin\theta \\ x_2 &= \dot{x}_d + \varepsilon\dot{\theta}\cos\theta \\ x_3 &= \theta \\ x_4 &= \dot{\theta}\ , \end{aligned} \qquad (5)$$

(3) and (4) are put into the following form:

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= -x_1 + \varepsilon \sin x_3 + F_d \\ \dot{x}_3 &= x_4 \\ \dot{x}_4 &= \frac{\varepsilon \cos x_3}{1-\varepsilon^2\cos^2 x_3}\left(x_1 - \varepsilon(1+x_4^2)\sin x_3 - F_d\right) \\ &\quad + \frac{1}{1-\varepsilon^2\cos^2 x_3} u \end{aligned} \qquad (6)$$

In the remainder of the paper, we assume that there is no disturbance, i.e. $F_d = 0$. It is further assumed that the translational and angular velocities are not available for feedback. That is to say, x_2 and x_4 are unmeasured. In previous work [12, 2, 6, 7], global stabilization is achieved by means of full-state feedback or partial-state feedback which requires the measurements of linear or angular velocities. In this paper, we will prove that the system (6) is asymptotically stabilizable at the equilibrium of all states zero using only the measurements of x_3 and/or x_1. The price to be paid in the process of this generalization is that we will assume an *a priori* bound on the initial conditions of the system state x is available for controller design. Nevertheless, this bound can be any finite positive constant.

More precisely, denoting $y := (x_1, x_3)$, our problem is formulated as follows.

Semi-Global Output Feedback Stabilization Problem. Given any compact set $\mathcal{S}_x \subset \mathbb{R}^4$, which is a neighborhood of the origin, there exists a dynamic output feedback control law of the form

$$\begin{aligned} \dot{\sigma} &= \nu(y, \sigma) \\ u &= \mu(y, \sigma) \end{aligned} \qquad (7)$$

and a compact set $\mathcal{S}_\sigma$ such that the equilibrium $(x, \sigma) = (0, 0)$ of the closed-loop system (6)-(7) is asymptotically stable, with domain of attraction containing $\mathcal{S}_x \times \mathcal{S}_\sigma$.

The interested reader is referred to [10] for a definition of semi-global output feedback stabilizability for general nonlinear systems. It is worth noting that the system (6) is not globally feedback linearizable [5] and does not satisfy the global Lipschitz condition because a quadratic nonlinearitiy appears in (6) in terms of the unmeasured state x_4. Therefore global output feedback stabilization algorithms proposed in the past literature (see [9] and references therein) are not applicable to system (6).

3 Semi-global Stabilization

This section is divided into two parts: we first consider the case where only the rotational angle θ is measured and considered as output. Then we deal with the case where both the angle and the position are used for controller design.

3.1 Using only angle measurements

In this section, the output is $x_3 = \theta$. A locally asymptotically stabilizing feedback law using only angle measurements was proposed in [7]. However, the stability region is small. The main purpose here is to achieve semi-global stabilization with a larger region of attraction.

As shown in [6], the following passivity-based feedback law

$$u = -k_1 x_3 - k_2 x_4\ , \quad k_1,\ k_2 > 0 \qquad (8)$$

makes the system (6) GAS (globally asymptotically stable), with respect to the storage function W defined by

$$W = \frac{1}{2}(x_1 - \varepsilon\sin x_3)^2 + \frac{1}{2}x_2^2 + \frac{k_1}{2}x_3^2 + \frac{1}{2}x_4^2(1-\varepsilon^2\cos^2 x_3)$$

However, (8) is a state-feedback controller and depends on the unmeasured state component x_4. The idea here is to introduce an observer to reconstruct x_4 and to prove that the observer-based output feedback controller $u = -k_1 x_3 - k_2\widehat{x}_4$ solves the semi-global output-feedback stabilization problem stated in Section 2.

Towards this end, let $\xi = x_4 - Lx_3$ where L is a positive constant. We have:

$$\begin{aligned} \dot{\xi} = -L\xi - L^2 x_3 - \frac{\varepsilon^2 \sin x_3 \cos x_3}{1-\varepsilon^2\cos^2 x_3} + \frac{1}{1-\varepsilon^2\cos^2 x_3} u \\ + \frac{1}{1-\varepsilon^2\cos^2 x_3}\left(\varepsilon x_1 \cos x_3 - \varepsilon^2 x_4^2 \sin x_3 \cos x_3\right) \end{aligned} \quad (9)$$

Introduce the following observer:

$$\dot{\widehat{\xi}} = -L\widehat{\xi} - L^2 x_3 - \frac{\varepsilon^2 \sin x_3 \cos x_3}{1-\varepsilon^2\cos^2 x_3} + \frac{1}{1-\varepsilon^2\cos^2 x_3} u \quad (10)$$

Setting $e = \widehat{\xi} - \xi$, we obtain:

$$\dot{e} = -Le - \frac{\varepsilon\cos x_3}{1-\varepsilon^2\cos^2 x_3}\left(x_1 - \varepsilon x_4^2 \sin x_3\right) \qquad (11)$$

Therefore, we establish the following.

Proposition 1 *The Semi-Global Output-Feedback Stabilization Problem is solvable for system (6) by using only θ measurements.*

Proof. We wish to prove that, for any compact set $\mathcal{S}_x \subset \mathbb{R}^4$, there exist a positive constant L_* and a compact set $\mathcal{S}_{\widehat{\xi}}$ such that for all $L > L_*$, the closed-loop system comprised of (6), (10) and

$$u \;=\; -k_1 x_3 - k_2 \widehat{x}_4 \tag{12}$$

where $\widehat{x}_4 = \widehat{\xi} + Lx_3$ and $k_1 \geq k_2 > 0$, is asymptotically stable at the origin with a basin of attraction containing $\mathcal{S}_x \times \mathcal{S}_{\widehat{\xi}}$.

Noticing that $\widehat{x}_4 = x_4 + e$ and $x = (x_1, \ldots, x_4)$, in coordinates (x, e), this closed loop system is expressed as:

$$\dot{x} \;=\; f(x, e) \tag{13}$$
$$\dot{e} \;=\; -Le + g(x) \tag{14}$$

where

$$f(x,e) \;=\; \begin{bmatrix} x_2 \\ -x_1 + \varepsilon \sin x_3 \\ x_4 \\ \Delta(x,e) \end{bmatrix} \tag{15}$$
$$g(x) \;=\; -\frac{\varepsilon \cos x_3}{1 - \varepsilon^2 \cos^2 x_3}\left(x_1 - \varepsilon x_4^2 \sin x_3\right) \tag{16}$$

with $\Delta(x,e) = \dfrac{1}{1-\varepsilon^2\cos^2 x_3}(-k_1 x_3 - k_2 x_4 - k_2 e + \varepsilon x_1 \cos x_3 - \varepsilon^2(1 + x_4^2)\sin x_3 \cos x_3)$.

It was shown in [6] that the zero equilibrium of the system $\dot{x} = f(x, 0)$ is GAS. By the Converse Lyapunov Theorem [4, Theorem 49.4], there exists a positive definite and proper Lyapunov function V_0 such that

$$\frac{\partial V_0}{\partial x}(x) f(x,0) \;\leq\; -\alpha(|x|) \tag{17}$$

where α is a class K-function (see [4, p. 7] for a definition).

In the following analysis, we will use the fact that $e(0)$ does not depend on L (cf. (19) and (23) below). This condition is guaranteed by choosing $\widehat{\xi}(0) = -Lx_3(0)$. In this case, $e(0) = -x_4(0)$.

Given any strictly positive real numbers c_0 and c_1, consider the Lyapunov function candidate V

$$V \;=\; c_0 \frac{V_0(x)}{c_0 + 1 - V_0(x)} \;+\; c_1 \frac{e^2}{c_1 + 1 - e^2} \tag{18}$$

For any $\rho > 0$, we will prove that there exists an $L_* > 0$ such that for all $L > L_*$ the time derivative of V along solutions of the closed loop system is negative definite for all initial conditions starting from the set:

$$\mathcal{S}_{xe} \;=\; \{(x, e) \,:\, \rho \leq V(x, e) \leq c_0^2 + c_1^2 + 1\} \tag{19}$$

Indeed, first note that V is well-defined on $\mathcal{S}_{xe}$. Then, differentiating V along the solutions of the system (6), (10) and (12) yields:

$$\begin{aligned} \dot{V} \;&=\; \frac{c_0(c_0+1)}{(c_0 + 1 - V_0(x))^2}\dot{V}_0 \;+\; \frac{2c_1(c_1+1)}{(c_1+1-e^2)^2} e\dot{e} \\ &\leq\; \frac{c_0(c_0+1)}{(c_0+1-V_0(x))^2}\Big[-\alpha(|x|) \\ &\qquad + \frac{\partial V_0}{\partial x}(x)(f(x,e) - f(x,0))\Big] \\ &\qquad + \frac{2c_1(c_1+1)}{(c_1+1-e^2)^2}(-Le^2 + eg(x)) \end{aligned} \tag{20}$$

Since $\mathcal{S}_{xe} \cap \{e = 0\}$ is a compact set, there exists a positive constant k_3 such that, for all x in the projection of $\mathcal{S}_{xe}$ on $\{e = 0\}$,

$$\left|\frac{\partial V_0}{\partial x}(x)(f(x,e) - f(x,0))\right| \leq k_3|e|\,, \quad |g(x)| \leq k_3 \tag{21}$$

Also, as shown in [10], we have:

$$\begin{aligned} \frac{c_0}{c_0+1} \;&\leq\; \frac{c_0(c_0+1)}{(c_0+1-V_0(x))^2} \leq \frac{(c_0^2+c_1^2+1+c_0)^2}{c_0(c_0+1)} \\ \frac{2c_1}{c_1+1} \;&\leq\; \frac{2c_1(c_1+1)}{(c_1+1-e^2)^2} \leq \frac{2(c_0^2+c_1^2+1+c_1)^2}{c_1(c_1+1)} \end{aligned}$$

as long as $V(x, e) \leq c_0^2 + c_1^2 + 1$.

Thus, (20) implies:

$$\begin{aligned} \dot{V} \leq &-\frac{c_0(c_0+1)}{(c_0+1-V_0(x))^2}\alpha(|x|) - L\frac{2c_1}{c_1+1}e^2 \\ &+ \underbrace{[\frac{k_3(c_0^2+c_1^2+1+c_0)^2}{c_0(c_0+1)} + \frac{2k_3(c_0^2+c_1^2+1+c_1)^2}{c_1(c_1+1)}]}_{a}|e| \end{aligned} \tag{22}$$

¿From (22), by applying Lemma 2.1 of [10], or by contradiction, there exists a positive constant L_* such that for all $L > L_*$, we have:

$$\dot{V} \;<\; 0 \quad \forall\, (x,e) \in \mathcal{S}_{xe} \setminus \{(0,0)\} \tag{23}$$

Therefore, from (23), it follows that all the solutions starting from $\mathcal{S}_{xe}$ under the constraint $e(0) = -x_4(0)$ are uniformly ultimately bounded and enter into the set $V \leq \rho$. Since $\rho > 0$ is arbitrary, the ultimate bound can be made arbitrarily small. To prove the asymptotic convergence of the solutions to the origin, it is sufficient to prove the asymptotic stability of the first approximation of system (13) and (14). If L is sufficiently large, since $k_1 \geq k_2 > 0$, the latter is proved because the following Jacobian matrix of the system (13) and (14)

$$\begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ -1 & 0 & \varepsilon & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ \frac{\varepsilon}{1-\varepsilon^2} & 0 & \frac{-k_1-\varepsilon^2}{1-\varepsilon^2} & \frac{-k_2}{1-\varepsilon^2} & \frac{-k_2}{1-\varepsilon^2} \\ \frac{-\varepsilon}{1-\varepsilon^2} & 0 & 0 & 0 & -L \end{bmatrix}$$

is asymptotically stable by means of the classical Routh-Hurwitz criterion. □

3.2 Using angle and position measurements

As pointed out in [6], in order to ensure a quick settling for the translational position, we have to use x_1 in the feedback law. The following state-feedback controller has been proposed in [6], instead of (8),

$$u = -k_0\varepsilon\cos x_3(-x_1+\varepsilon\sin x_3) - k_1 x_3 - k_2 x_4 \quad (24)$$

which globally asymptotically stabilizes the RTAC system with respect to the storage function:

$$W_a = \frac{k_0+1}{2}[(x_1-\varepsilon\sin x_3)^2 + x_2^2] + \frac{k_1}{2}x_3^2 + \frac{1}{2}x_4^2(1-\varepsilon^2\cos^2 x_3)$$

Analogously, using the observer (10) and setting $\widehat{x}_4 = \widehat{\xi} + Lx_3$, the output feedback law derived from (24) with x_4 replaced by $\widehat{x}_4$ is a semi-globally stabilizing controller. The proof is similar to the proof of Proposition 1. As the simulations show (see Figures 2 and 3), the output-feedback controller using output (x_1, x_3)

$$u = -k_0\varepsilon\cos x_3(-x_1+\varepsilon\sin x_3) - k_1 x_3 - k_2 \widehat{x}_4 \quad (25)$$

results in a smaller settling time for the translational motion x_1 than the previous one (12) using only x_3 measurements.

In fact, when $y = (x_1, x_3)$ is considered as output, we can prove the following general result for the RTAC system.

Proposition 2 *For any GAS and LES (locally exponentially stabilizing) state-feedback controller $\overline{u}(x)$ for system (6), there exists always an observer-based output feedback controller $\overline{u}(x_1, \widehat{x}_2, x_3, \widehat{x}_4)$, with*

$$\begin{aligned} \dot{\widehat{\zeta}} &= \chi_1(\widehat{\zeta}, y), \quad \widehat{x}_2 = \widehat{\zeta} + Lx_1 && (26)\\ \dot{\widehat{\xi}} &= \chi_2(\widehat{\xi}, y), \quad \widehat{x}_4 = \widehat{\xi} + Lx_3 && (27)\end{aligned}$$

which solves the Semi-Global Output-Feedback Stabilization Problem for system (6).

Proof. In addition to the above observer (10) for the reconstruction of the unmeasured state x_4, we need to build an observer for the unmeasured state x_2. Letting $\zeta = x_2 - Lx_1$, we have:

$$\dot{\zeta} = -L\zeta - L^2x_1 - x_1 + \varepsilon\sin x_3 \quad (28)$$

Introducing an observer as:

$$\dot{\widehat{\zeta}} = -L\widehat{\zeta} - L^2x_1 - x_1 + \varepsilon\sin x_3 \quad (29)$$

and denoting $e_2 = \widehat{\zeta} - \zeta$, it holds:

$$\dot{e}_2 = -Le_2 \quad (30)$$

Letting $\overline{e} = (e, e_2)^T$, the closed loop system (6) with $u = \overline{u}(x_1, \widehat{x}_2, x_3, \widehat{x}_4)$, where $\widehat{x}_2 = \widehat{\zeta} + Lx_1$ and $\widehat{x}_4 = \widehat{\xi} + Lx_3$ as defined in the subsection 3.1, is rewritten as:

$$\begin{aligned} \dot{x} &= \overline{f}(x, \overline{e}) && (31)\\ \dot{\overline{e}} &= -L\overline{e} + \overline{g}(x) && (32)\end{aligned}$$

where

$$\overline{f}(x,\overline{e}) = \begin{bmatrix} x_2 \\ -x_1+\varepsilon\sin x_3 \\ x_4 \\ \kappa(x,e)\end{bmatrix} \quad (33)$$

$$\overline{g}(x) = \begin{bmatrix} -\dfrac{\varepsilon\cos x_3}{1-\varepsilon^2\cos^2 x_3}\left(x_1 - \varepsilon x_4^2\sin x_3\right) \\ 0 \end{bmatrix} \quad (34)$$

where

$$\begin{aligned}\kappa(x,e) = {} & \frac{\varepsilon\cos x_3}{1-\varepsilon^2\cos^2 x_3}(x_1 - \varepsilon(1+x_4^2)\sin x_3) \\ & + \frac{1}{1-\varepsilon^2\cos^2 x_3}\overline{u}(x_1, x_2+e_2, x_3, x_4+e)\ .\end{aligned}$$

The proof for the semi-global stabilizability of (31) and (32) follows the same lines as the proof of Proposition 1. Again, these techniques were used in previous work on issues related to semi-global output feedback stabilization (see, e.g., [10, 8]). □

3.3 Simulation results

Figure 2 gives the simulation for the normalized position x_1 and the control torque u with the output feedback control law (12) whereas Figure 3 gives corresponding results with the output feedback control law (25). In these simulations we chose the following values for initial conditions and design parameters:

$$\begin{array}{l} x(0) = (1,1,1,1)\ , \quad e(0) = -1\ , \quad L = 10\ , \\ k_0 = 10\ , \quad k_1 = 1\ , \quad k_2 = 0.5\ , \quad \varepsilon = 0.1\ . \end{array} \quad (35)$$

We observe that as in [6] with state-feedback the output feedback control law (25) yields faster convergence for x_1 than the output feedback control law (12) based only on the measurement of θ. Also, according to our simulations, we observe that the output-feedback control laws proposed in this paper ensure performance as good as the passivity-based state-feedback controllers in [6] when the design parameter L is chosen sufficiently large.

4 Robustness Issue

A practically meaningful controller must keep stability properties even in the presence of uncertainties. Consider the more realistic situation where $F_d \neq 0$ in the original system (1), or (3). Then, the closed-loop system (31)-(32) is rewritten as

$$\begin{aligned} \dot{x} &= \widetilde{f}(x,\overline{e}) && (36)\\ \dot{\overline{e}} &= -L\overline{e} + \widetilde{g}(x) && (37)\end{aligned}$$

where

$$\begin{aligned} \widetilde{f}(x,\overline{e}) &= \overline{f}(x,\overline{e}) + (0,\,1,\,0,\,\tfrac{-\varepsilon\cos x_3}{1-\varepsilon^2\cos^2 x_3})^T F_d && (38)\\ \widetilde{g}(x) &= \overline{g}(x) - (\tfrac{-\varepsilon\cos x_3}{1-\varepsilon^2\cos^2 x_3},\,1)^T F_d && (39)\end{aligned}$$

It was shown in Section 3 that the disturbance free system (36)-(37) (i.e. $F_d = 0$) is asymptotically stable where the domain of attraction increases as L does. A classical theorem on "total stability" [4] asserts that a small disturbance F_d yields bounded solutions.

For simulation use, we take $F_d = 0.1 \sin t$ and consider the output-feedback controller (25) so that (37) is simplified to:

$$\dot{e} = -Le + \widetilde{g}_1(x) \qquad (40)$$

Figure 4 illustrates the trajectories of x_1 and x_3 with the same choice of initial conditions and design constants in (35) and shows that the feedback law (25) achieves better performance than with no control input (i.e. $u = 0$), as pointed out in [12].

5 Conclusion

We have revisited in this paper the stabilization problem for the nonlinear benchmark example. A semi-global output feedback approach is proposed using the measurements of the angle and/or the position of the system. This complements the previous work in this topic with full-state feedback or partial-state feedback but using the measurements of linear or angular velocities. It is worth noting that the semi-global asymptotic stabilization obtained is significant from an engineering point of view since any control system should eventually have the global boundedness property for the solutions.

Acknowledgements: This work was supported by the ARC Large Grant Reference No. A49530078. Z. P. Jiang thanks Andrew Teel for helpful comments on results in [10]. We are grateful to an anonymous reviewer for a number of useful remarks.

References

[1] A. Bacciotti, Linear feedback: the local and potentially global stabilization of cascade systems, *Prep. IFAC NOLCOS'92*, Bordeaux, pp. 21-25, (1992).

[2] R. T. Bupp, D. S. Bernstein and V. T. Coppola, A benchmark problem for nonlinear control design, *Proc. American Control Conf.*, Seattle, Washington, (1995).

[3] G. Escobar and R. Ortega, Output-feedback global stabilization of a nonlinear benchmark system using a saturated passivity-based controller. Preprint. (1997).

[4] W. Hahn, *Stability of Motion.* Berlin: Springer-Verlag, (1967).

[5] A. Isidori, *Nonlinear Control Systems.* 3rd Edition, London: Springer-Verlag, (1995).

[6] M. Jankovic, D. Fontaine and P. V. Kokotović, TORA example: cascade- and passivity-based control designs, *IEEE Trans. Control Systems Technology*, **4**, pp. 292-297, (1996).

[7] I. Kanellakopoulos and J. Zhao, Tracking and disturbance rejection for the benchmark nonlinear control problem, *Proc. American Control Conf.*, pp. 4360-4362, Seattle, Washington, (1995).

[8] H. K. Khalil and F. Esfandiari, Semiglobal stabilization of a class of nonlinear systems using output feedback, *IEEE Trans. Automat. Control*, **38**, pp. 1412-1415, (1993).

[9] L. Praly and Z. P. Jiang, Stabilization by output feedback for systems with ISS inverse dynamics, *Systems & Control Letters*, **21**, 19-33, (1993).

[10] A. Teel and L. Praly, Tools for semi-global stabilization by partial state and output feedback, *SIAM J. Control Optimization*, **33**, pp. 1443-1488, (1995).

[11] P. Tsiotras, M. Corless and M. A. Rotea, An $\mathcal{L}_2$ disturbance attenuation approach to the nonlinear benchmark problem, *Proc. American Control Conf.*, pp. 4352-4356, Seattle, Washington, (1995).

[12] C.-J. Wan, D. S. Bernstein and V. T. Coppola, Global stabilization of the oscillating eccentric rotor, *Proc. 33rd IEEE Conf. Dec. Control*, pp. 4024-4029, Lake Buena Vista, (1994).

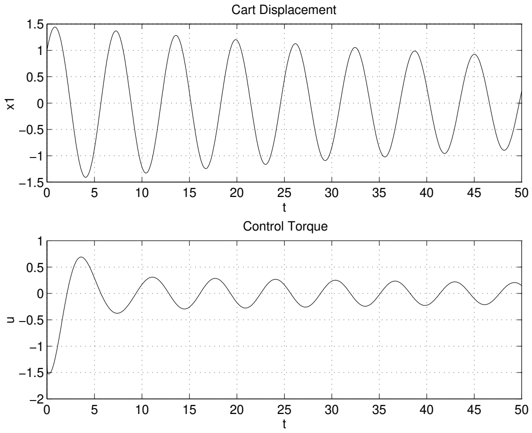

Figure 2: Output feedback controller (12), $F_d = 0$

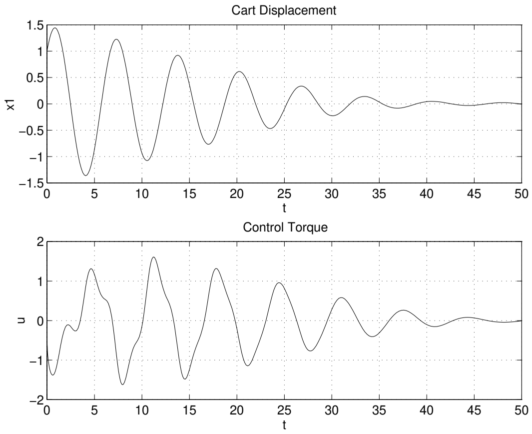

Figure 3: Output feedback controller (25), $F_d = 0$

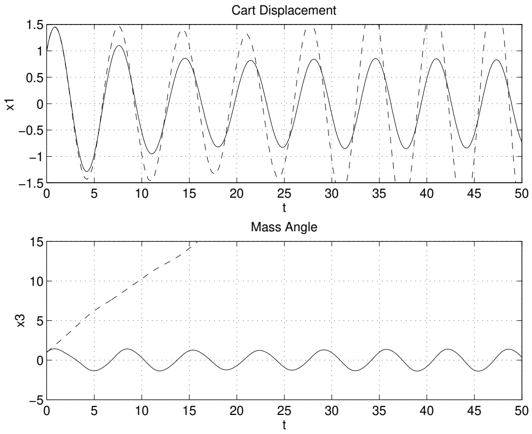

Figure 4: Output feedback controller (25), $F_d = 0.1 \sin t$. The dashed lines correspond to the uncontrolled case.