DECENTRALIZED ROBUST DISTURBANCE ATTENUATION FOR LARGE-SCALE NONLINEAR SYSTEMS

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Abstract: We solve the problem of decentralized H_∞ almost disturbance decoupling for a class of large-scale nonlinear uncertain systems in the absence of matching conditions. The method combines ideas from decentralized adaptive control and centralized nonlinear H_∞ control. We relax earlier assumptions on the uncertain time-varying interconnections which are demanded to be only bounded by general nonlinear functions in this work.

Keywords: Large-scale systems; nonlinear systems; decentralized control; disturbance attenuation; Lyapunov methods, backstepping.

1. INTRODUCTION

We consider in this paper the decentralized robust H_∞ control problem for a class of large-scale nonlinear systems motivated by (Jain and Khorrami, 1997). Uncertain interconnections are extended to be time-varying and be bounded by general nonlinear functions. Disturbances enter each subsystem affinely with subsystem states interacting through their input vector fields. We recursively construct a decentralized control law and the internally stable closed-loop system maintains any given $\mathcal{L}_2$-gain from the disturbance input to the output. Our results are intended to make up the gap between decentralized and centralized H_∞ control.

The notation used in this paper is standard. $|\cdot|$ denotes the usual Euclidean norm for vectors. $\bigtriangledown f$ denotes the gradient $\frac{\partial f}{\partial s}$ of a real-valued function f. We say that $z : (0,T) \to \Re^k$ is in $\mathcal{L}_2(0,T)$ if $\int_0^T |z(t)|^2 dt < \infty$.

2. PROBLEM FORMULATION

We consider in this paper a class of large-scale nonlinear systems S which are composed of the following single-input single-output (SISO) subsystems $S_i (1 \le i \le N)$:

$$
\begin{aligned}
\begin{bmatrix} \dot z_{i1} \\ \vdots \\ \dot z_{i,\kappa-1} \\ \dot z_{i\kappa} \end{bmatrix} &= \begin{bmatrix} 0 & 1 & & 0 \\ & & \ddots & \\ 0 & 0 & & 1 \\ 0 & 0 & \dots & 0 \end{bmatrix} \begin{bmatrix} z_{i1} \\ \vdots \\ z_{i,\kappa-1} \\ z_{i\kappa} \end{bmatrix} + \begin{bmatrix} 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix} \\
&\quad \cdot \{\xi_{i1} + \gamma_{i0}(t,z) + p_{i0}(t,z)\omega_i\} \\
&= A_i z_i + B_i \left[\xi_{i1} + \gamma_{i0}(t,z) + p_{i0}(t,z)\omega_i\right] \\
\dot\xi_{i1} &= \xi_{i2} + \gamma_{i1}(t,z,\xi_{i1}) + p_{i1}(t,z,\xi_{i1})\omega_i \\
&\ \ \vdots \\
\dot\xi_{i,n-\kappa} &= u_i + \gamma_{i,n-\kappa}(t,z,\xi_i) + p_{i,n-\kappa}(t,z,\xi_i)\omega_i \\
y_i &= z_{i1}
\end{aligned}
\tag{1}
$$

where $z_i = (z_{i1}, \dots, z_{i\kappa}) \in \Re^\kappa$, $z = (z_1^T, \dots, z_N^T)^T$, $\xi_i = (\xi_{i1}, \dots, \xi_{i,n-\kappa}) \in \Re^{n-\kappa}$, and $\xi = (\xi_1^T, \dots, \xi_N^T)^T$, z and ξ are the state vectors, $u_i \in \Re$ is the control input, $\omega_i \in \Re^{m_i}$ is the disturbance input, $y_i \in \Re$ is the to-be-controlled output; the unknown functions $\gamma_{il},\ p_{il},\ 0 \le l \le n-\kappa$ are locally Lipschitz in states and piecewise continuous in t, and $\gamma_{il}(t,0,0,\dots,0) = 0$.

Our class of systems (1) is motivated by the time-invariant decentralized strict feedback form of (Jain and Khorrami, 1997) in which necessary and sufficient geometric conditions are given to characterize a subclass of systems (1). Nevertheless, we allow in this paper for a larger set of nonlinear interconnections, as shown in the following assumptions.

Assumption 1. There exist known smooth functions $a_{ilj}(\cdot), b_{ilj}(\cdot), \varphi_{ilj}(\cdot), \Phi_{ilj}(\cdot)$, with $\varphi_{ilj}(0) = \Phi_{ilj}(0) = 0$, such that for each $0 \le l \le n-k$,

$$
\begin{aligned}
&|\gamma_{il}(t,z,\xi_{i1},\ldots,\xi_{il}) - \gamma_{il}(t,0,\xi_{i1},\ldots,\xi_{il})| \\
&\le \sum_{j=1}^{N} a_{ilj}(\xi_{i1},\ldots,\xi_{il})\varphi_{ilj}(|z_j|), \qquad (2) \\
&|p_{il}(t,z,\xi_{i1},\ldots,\xi_{il}) - p_{il}(t,0,\xi_{i1},\ldots,\xi_{il})| \\
&\le \sum_{j=1}^{N} b_{ilj}(\xi_{i1},\ldots,\xi_{il})\Phi_{ilj}(|z_j|) \qquad (3)
\end{aligned}
$$

By convention, let $a_{i0j} = b_{i0j} = 1$.

Assumption 2. There exist smooth functions $\psi_{il}(\cdot)$, $\Psi_{il}(\cdot)$, with $\psi_{il}(0) = 0$, such that for each $0 \le l \le n-k$,

$$
\begin{aligned}
|\gamma_{il}(t,0,\xi_{i1},\ldots,\xi_{il})| &\le \psi_{il}(|(\xi_{i1},\ldots,\xi_{il})|), \qquad (4) \\
|p_{il}(t,0,\xi_{i1},\ldots,\xi_{il})| &\le \Psi_{il}(|(\xi_{i1},\ldots,\xi_{il})|). \qquad (5)
\end{aligned}
$$

Since $\gamma_{i0}(t,0) = 0$, $\psi_{i0} \equiv 0$; and Ψ_{i0} is a nonnegative constant.

The objective of our design is to find robust decentralized controllers in terms of local subsystem states and make the whole interconnected system globally uniformly asymptotically stable (GUAS) while arbitrarily attenuating the effect of the disturbance. A precise statement of this control problem is given below:

Problem of Decentralized H_∞ Almost Disturbance Decoupling: Find decentralized smooth state feedback controllers $u_i = u_i(z_i, \xi_i)$ such that, for any given positive constant μ, the closed-loop interconnected system satisfies the following dissipation inequality

$$
\int_0^T |y|^2 dt \le \mu \int_0^T |w|^2 dt + \nu(z(0),\xi(0)) \qquad (6)
$$
$$
\forall \omega \in \mathcal{L}_2(0,T), \forall T \ge 0
$$

where ν is a positive semidefinite function and $(z(0),\xi(0))$ is the initial condition. Furthermore, the origin is GUAS if $w = 0$.

3. MAIN RESULT

The following theorem states our main result:

Theorem 1. Under Assumption 1 and Assumption 2, the Problem of Decentralized H_∞ Almost Disturbance Decoupling is solvable for system (1).

The proof of Theorem 1 uses a combination of the centralized H_∞ almost disturbance decoupling (Marino *et al.*, 1994)–and its robust version (Jiang and Jiang, 1997)–and decentralized designs (Han and Chen, 1995; Jain and Khorrami, 1997). A stepwise procedure is presented by application of robust backstepping to the large-scale system (1).

Proof: We start by considering the z_i-subsystem with ξ_{i1} as the virtual control input. Choose the Lyapunov function candidate:

$$
V_i(z_i) = \phi_i\left(V_{i0}(z_i)\right) \qquad (7)
$$

where ϕ_i is a smooth $\mathcal{K}_\infty$ function and $V_{i0} = z_i^T P_i z_i$ with $P_i > 0$ solving the algebraic Riccati equation:

$$
A_i^T P_i + P_i A_i - 2\varepsilon_i P_i B_i B_i^T P_i + Q_i = 0 \qquad (8)
$$

with $\varepsilon_i > 0$, and $Q_i > 0$.

Differentiating (7) along the solution of z_i-subsystem, we have

$$
\begin{aligned}
\dot V_i = \frac{\partial \phi_i}{\partial V_{i0}} 2 z_i^T P_i \{A_i z_i + B_i [\xi_{i1} + (\gamma_{i0}(t,z) - \gamma_{i0}(t,0)) \\
+ (p_{i0}(t,z) - p_{i0}(t,0))\omega_i + p_{i0}(t,0)\omega_i]\} \qquad (9)
\end{aligned}
$$

For the interconnected terms we apply Assumption 1 and Cauchy-Schwartz inequality to decouple the interactions:

$$
\begin{aligned}
&\frac{\partial \phi_i}{\partial V_{i0}} 2 z_i^T P_i B_i (\gamma_{i0}(t,z) - \gamma_{i0}(t,0)) \qquad (10) \\
&\le |\frac{\partial \phi_i}{\partial V_{i0}}|^2 |z_i^T P_i B_i|^2 \sum_{j=1}^{N} \delta_{j0} + \sum_{j=1}^{N} \delta_{j0}^{-1} \varphi_{i0j}^2(|z_j|)\ , \\
&\frac{\partial \phi_i}{\partial V_{i0}} 2 z_i^T P_i B_i (p_{i0}(t,z) - p_{i0}(t,0))\omega_i \qquad (11) \\
&\le \sum_{j=1}^{N} \frac{1}{\tau_i} |\frac{\partial \phi_i}{\partial V_{i0}}|^2 |z_i^T P_i B_i|^2 \Phi_{i0j}^2(|z_j|) + N\tau_i|\omega_i|^2 \\
&\le \frac{1}{4\tau_i^2} |\frac{\partial \phi_i}{\partial V_{i0}}|^4 |z_i^T P_i B_i|^4 \sum_{j=1}^{N} \varrho_{j0} \\
&\quad + \sum_{j=1}^{N} \varrho_{j0}^{-1} \Phi_{i0j}^4(|z_j|) + N\tau_i|\omega_i|^2\ ,
\end{aligned}
$$

where $\delta_{j0}, \varrho_{i0}$ are positive design parameters.

Apply Assumption 2,

$$
\begin{aligned}
&\frac{\partial \phi_i}{\partial V_{i0}} 2 z_i^T P_i B_i p_{i0}(t,0)\omega_i \qquad (12) \\
&\le \frac{1}{d_{2i}} |\frac{\partial \phi_i}{\partial V_{i0}}|^2 |z_i^T P_i B_i|^2 \Psi_{i0}^2 + d_{2i}|\omega_i|^2 \qquad (13)
\end{aligned}
$$

Choose our virtual control $\xi_{i1} = \xi^*_{i1}(z_i)$ as

$$\xi^*_{i1}(z_i) = -\left\{\varepsilon_i B_i^T P_i z_i + \frac{1}{2}\frac{\partial \phi_i}{\partial V_{i0}}(z_i^T P_i B_i)\left(\sum_{j=1}^{N}\delta_{j0}\right)\right.$$
$$+\frac{1}{8\tau_i}\left(\frac{\partial \phi_i}{\partial V_{i0}}\right)^3 (z_i^T P_i B_i)^3 \left(\sum_{j=1}^{N}\varrho_{j0}\right)$$
$$\left. + \frac{1}{2d_{2i}}\left(\frac{\partial \phi_i}{\partial V_{i0}}\right)(z_i^T P_i B_i)\Psi^2_{i0}\right\} \quad (14)$$

When substituting (14) into (9), it holds:

$$\dot{V}_i \leq \frac{\partial \phi_i}{\partial V_{i0}}\left(-z_i^T Q_i z_i\right) + \left[\sum_{j=1}^{N}\delta_{i0}^{-1}\varphi^2_{i0j}(|z_j|)\right.$$
$$\left.+\sum_{j=1}^{N}\varrho_{j0}^{-1}\Phi^4_{i0j}(|z_j|)\right] + (N\tau_i + d_{2i})|\omega_i|^2$$
$$\triangleq -\alpha_{i0}(|z_i|) + \sum_{j=1}^{N}\beta_{i0j}(|z_j|) + \gamma_{i0}|\omega_i|^2 \quad (15)$$

We leave the choice of ϕ_i till the design problem is completed. From here, we start our recursive backstepping procedure.

Step 1: Augment the z_i-system with the ξ_{i1}-subsystem, and choose a Lyapunov function as

$$W_{i1}(z_i, \xi_{i1}) = V_i(z_i) + (\xi_{i1} - \xi^*_{i1})^2 \quad (16)$$

Note

$$-\dot{\xi}^*_{i1} = -\frac{\partial \xi^*_{i1}}{\partial z_i}(A_i z_i + B_i \xi_{i1}) - \frac{\partial \xi^*_{i1}}{\partial z_i}B_i\left[\gamma_{i0}(t,z)\right.$$
$$\left.+p_{i0}(t,z)\omega_i\right]$$
$$\triangleq \vartheta_{i1}(z_i,\xi_{i1}) + \sigma_{i0}[\gamma_{i0}(t,z) + p_{i0}(t,z)\omega_i] \quad (17)$$

Differentiating W_{i1} along the solutions of the (z_i, ξ_{i1})-subsystem yields

$$\dot{W}_{i1} \leq -\alpha_{i0}(|z_i|) + \sum_{j=1}^{N}\beta_{i0j}(|z_j|) + \gamma_{i0}|\omega_i|^2$$
$$+2\widetilde{\xi}_{i1}\left\{\xi_{i2} + \left(\frac{\partial \phi_i}{\partial V_{i0}}z_i^T P_i B_i + \vartheta_{i1}\right)\right.$$
$$+\sum_{k=0}^{1}\sigma_{ik}[\gamma_{ik}(t,z,\xi_{ik}) - \gamma_{ik}(t,0,\xi_{ik})] \quad (18)$$
$$+\sum_{k=0}^{1}\sigma_{ik}[p_{ik}(t,z,\xi_{ik}) - p_{ik}(t,0,\xi_{ik})]\omega_i$$
$$\left.+\gamma_{i1}(t,0,\xi_{i1}) + \sum_{k=0}^{1}\sigma_{ik}p_{ik}(t,0,\xi_{ik})\omega_i\right\}$$

where we denote $\widetilde{\xi}_{i1} = \xi_{i1} - \xi^*_{i1}$ and $\sigma_{i1} = 1$.

To handle the time-varying terms in (18), use bounds from Assumptions 1 and 2.

Since ξ^*_{i1} and ψ_{i1} are smooth and are zero at zero, there exist smooth functions $\widetilde{\psi}_{i1}$, such that,

$$2\widetilde{\xi}_{i1}\gamma_{i1}(t,0,\xi_{i1}) \quad (19)$$
$$\leq 2|\widetilde{\xi}_{i1}|\widetilde{\psi}_{i1}(z_i,\xi_{i1})\left(|\widetilde{\xi}_{i1}| + |z_i|\right)$$
$$\leq 2|\widetilde{\xi}_{i1}|^2\widetilde{\psi}_{i1}(z_i,\xi_{i1}) + 2|\widetilde{\xi}_{i1}|\cdot|z_i|\cdot\widetilde{\psi}_{i1}(z_i,\xi_{i1})$$
$$\leq 2|\widetilde{\xi}_{i1}|^2\widetilde{\psi}_{i1}(z_i,\xi_{i1}) + \frac{1}{d_{1i}}|\widetilde{\xi}_{i1}|^2\widetilde{\psi}^2_{i1}(z_i,\xi_{i1}) + d_{1i}|z_i|^2$$

where $d_{1i} > 0$. Similarly,

$$2\widetilde{\xi}_{i1}\sum_{k=0}^{1}\sigma_{ik}p_{ik}(t,0,\xi_{ik})\omega_i$$
$$\leq \frac{1}{d_{2i}}|\widetilde{\xi}_{i1}|^2\left[\sum_{k=0}^{1}\sigma_{ik}\Psi_{ik}(|\xi_{ik}|)\right]^2 + d_{2i}|\omega_i|^2 \;, \quad (20)$$
$$2\widetilde{\xi}_{i1}\sum_{k=0}^{1}\sigma_{ik}[\gamma_{ik}(t,z,\xi_{ik}) - \gamma_{ik}(t,0,\xi_{ik})]$$
$$\leq |\widetilde{\xi}_{i1}|^2\sum_{k=0}^{1}|\sigma_{ik}|^2\sum_{j=1}^{N}\delta_{jk}a^2_{ikj}(\xi_{ik})$$
$$+\sum_{k=0}^{1}\sum_{j=1}^{N}\delta_{jk}^{-1}\varphi^2_{ikj}(|z_j|)\;, \quad (21)$$
$$2\widetilde{\xi}_{i1}\sum_{k=0}^{1}\sigma_{ik}(z_i)[p_{ik}(t,z) - p_{ik}(t,0,\xi_{ik})]\omega_i$$
$$\leq \frac{1}{4\tau_i^2}|\widetilde{\xi}_{i1}|^4\sum_{k=0}^{1}|\sigma_{ik}|^4\cdot\sum_{j=1}^{N}\varrho_{jk}b^4_{ikj}(\xi_{ik})$$
$$+\sum_{k=0}^{1}\sum_{j=1}^{N}\varrho_{jk}^{-1}\Phi^4_{ikj}(|z_j|) + N\tau_i|\omega_i|^2\;, \quad (22)$$

where $d_{2i}, \delta_{ik}, \varrho_{ik} > 0$.

Now choose the virtual controller $\xi_{i2} = \xi^*_{i2}$ as

$$\xi^*_{i2} = -\frac{1}{2}\left\{c_{i1}\widetilde{\xi}_{i1} + \frac{\partial \phi_i}{\partial V_{i0}}2z_i^T P_i B_i + 2\vartheta_{i1}\right.$$
$$+2\widetilde{\xi}_{i1}\widetilde{\psi}_{i1}(z_i,\xi_{i1}) + \frac{1}{d_{i1}}\widetilde{\xi}_{i1}\widetilde{\psi}^2_{i1}(z_i,\xi_{i1})$$
$$+\frac{1}{d_{2i}}\widetilde{\xi}_{i1}\left[\sum_{k=0}^{1}\sigma_{ik}\Psi_{ik}(|\xi_{ik}|)\right]^2$$
$$+\widetilde{\xi}_{i1}\sum_{k=0}^{1}\sigma^2_{ik}\sum_{j=1}^{N}\delta_{jk}a^2_{ikj}(\xi_{ik})$$
$$\left.+\frac{1}{4\tau_i^2}\widetilde{\xi}^3_{i1}\sum_{k=0}^{1}\sigma^4_{ik}\sum_{j=1}^{N}\varrho_{jk}b^4_{ikj}(\xi_{ik})\right\} \quad (23)$$

with $c_{i1} > 0$. Then (18) with the bounds (19)-(22) implies:

$$\dot{W}_{i1} \leq -\alpha_{i1}(|z_i|) - C_{i11}\widetilde{\xi}_{i1}^2 + \sum_{j=1}^{N} \beta_{ij}(|z_j|) + \gamma_{i1}|\omega_i|^2 \tag{24}$$

where

$$\alpha_{i1}(|z_i|) = \alpha_{i0}(|z_i|) - d_{1i}|z_i|^2 \tag{25}$$
$$\beta_{i1j}(|z_j|) = \sum_{k=0}^{1} \left\{ \delta_{jk}^{-1}\varphi_{ikj}^2(|z_j|) + \varrho_{jk}^{-1}\Phi_{ikj}^4(|z_j|) \right\} + \beta_{i0j} \tag{26}$$
$$\gamma_{i1} = \gamma_{i0} + N\tau_i + d_{2i} \tag{27}$$
$$C_{i11} = c_{i1} \tag{28}$$

Step l ($2 \leq l \leq n - \kappa$): The proof of the following Claim is given in the Appendix.

Claim: If from the previous step the following inequality holds:

$$\dot{W}_{i,l-1} \leq -\alpha_{i,l-1}(|z_i|) - \sum_{k=1}^{l-1} C_{i,l-1,k}\widetilde{\xi}_{ik}^2 + \sum_{j=1}^{N} \beta_{i,l-1,j}(|z_j|) + \gamma_{i,l-1}|\omega_i|^2 \tag{29}$$

then by constructing the decentralized virtual control ξ_{il}^*, we can obtain

$$\dot{W}_{il} \leq -\alpha_{il}(|z_i|) - \sum_{k=1}^{l} C_{ilk}\widetilde{\xi}_{ik}^2 + \sum_{j=1}^{N} \beta_{ilj}(|z_j|) + \gamma_{il}|\omega_i|^2. \tag{30}$$

where

$$\alpha_{il}(|z_i|) = \alpha_{i,l-1}(|z_i|) - d_{1i}|z_i|^2 \ , \tag{31}$$
$$\beta_{ilj}(|z_j|) = \sum_{k=0}^{l} \left\{ \delta_{jk}^{-1}\varphi_{ikj}^2(|z_j|) + \varrho_{jk}^{-1}\Phi_{ikj}^4(|z_j|) \right\} + \beta_{i,l-1,j} \ , \tag{32}$$
$$\gamma_{il} = \gamma_{i,l-1} + N\tau_i + d_{2i} \ , \tag{33}$$
$$C_{ilk} = \begin{cases} C_{i,l-1,k} - d_{3i} & k = 1, \ldots, l-1 \\ c_{ik} \quad \text{for} & k = l \end{cases} \tag{34}$$

and

$$W_{il} = W_{i,l-1} + (\xi_{il} - \xi_{il}^*(z_i, \xi_{i1}, \ldots, \xi_{i,l-1}))^2 \tag{35}$$

By construction, true controller $u_i = \xi_{i,n-\kappa}^*(z_i, \xi_i)$. □

At Step $n - \kappa$, we obtain the following dissipation inequality for system S_i associated with storage function $W_i \triangleq W_{i,n-\kappa}$:

$$\dot{W}_i \leq -\alpha_i(|z_i|) - \sum_{l=1}^{n-\kappa} C_{il}\widetilde{\xi}_{il}^2 + \sum_{j=1}^{N} \beta_{ij}(|z_j|) + \gamma_i|\omega_i|^2 \tag{36}$$

where

$$\alpha_i(|z_i|) = \frac{\partial \phi_i}{\partial V_{i0}} \left(z_i^T Q_i z_i \right) - (n-\kappa)d_{1i}|z_i|^2 \ , \tag{37}$$
$$\beta_{ij}(|z_j|) = \sum_{l=0}^{n-\kappa} \left\{ (n-\kappa-l+1)\delta_{jl}^{-1}\varphi_{ilj}^2(|z_j|) + (n-\kappa-l+1)\varrho_{jl}^{-1}\Phi_{ilj}^4(|z_j|) \right\} \ , \tag{38}$$
$$\gamma_i = (n-\kappa+1)(N\tau_i + d_{2i}) \ , \tag{39}$$
$$C_{il} = c_{il} - (n-\kappa-l)d_{3i} \ . \tag{40}$$

Choose $d_{3i} < c_{il}/(n-\kappa-l)$ so that $C_{il} > 0$.

Define a Lyapunov function for the whole interconnected system S as $W = \sum_{i=1}^{N} W_i(z_i, \xi_i)$. Then we will compensate for effect of other subsystem interactions using the subsystem stability margin.

From (36), we know

$$\dot{W} \leq \sum_{i=1}^{N} \left\{ -\alpha_i(|z_i|) - \sum_{l=1}^{n-\kappa} C_{il}\widetilde{\xi}_{il}^2 + \sum_{j=1}^{N} \beta_{ij}(|z_j|) + \gamma_i|\omega_i|^2 \right\} \tag{41}$$

Notice $\sum_{i=1}^{N}\sum_{j=1}^{N} \beta_{ij}(|z_j|) = \sum_{i=1}^{N}\sum_{j=1}^{N} \beta_{ji}(|z_i|)$.

For smooth functions vanishing at the origin $\varphi_{jli}(\cdot)$ and $\Phi_{jli}(\cdot)$, we have

$$\varphi_{jli}(|z_i|) = |z_i| \int_0^1 \nabla\varphi_{jli}(s|z_i|)ds \leq |z_i|\widetilde{\varphi}_{jli}(|z_i|) \ ,$$
$$\Phi_{jli}(|z_i|) = |z_i| \int_0^1 \nabla\Phi_{jli}(s|z_i|)ds \leq |z_i|\widetilde{\Phi}_{jli}(|z_i|) \ ,$$

where $\widetilde{\varphi}_{jli}(|z_i|), \widetilde{\Phi}_{jli}(|z_i|)$ are smooth and nondecreasing functions.

Observe that $\lambda_{\min}(P_i)|z_i|^2 \leq V_{i0}(z_i)$ which gives

$$|z_i| \leq \sqrt{\frac{V_{i0}(z_i)+1}{\lambda_{\min}(P_i)}} \triangleq \eta_i(V_{i0}) \tag{42}$$

Note that $\eta_i^{,}s$ are smooth positive functions.

By the nondecreasing property of functions $\widetilde{\varphi}_{jli}(\cdot)$ and $\widetilde{\Phi}_{jli}(\cdot)$, we have

$$\widetilde{\varphi}_{jli}(|z_i|) \leq \widetilde{\varphi}_{jli} \circ \eta_i(V_{i0}) \ , \quad \widetilde{\Phi}_{jli}(|z_i|) \leq \widetilde{\Phi}_{jli} \circ \eta_i(V_{i0}) \ . \tag{43}$$

Construct the derivative of $\phi_i(V_{i0})$ as

$$\frac{\partial \phi_i}{\partial (V_{i0})} = k_i + \frac{1}{\lambda_{\min}(Q_i)} \sum_{j=1}^{N} \sum_{l=0}^{n-\kappa} (n-\kappa-l+1) \cdot$$
$$\delta_{il}^{-1} \left[\widetilde{\varphi}_{jli} \circ \eta_i(V_{i0})\right]^2 + \frac{V_{i0}}{\lambda_{\min}(Q_i) \cdot \lambda_{\min}(P_i)} \cdot$$
$$\sum_{j=1}^{N} \sum_{l=0}^{n-\kappa} (n-\kappa-l+1) \varrho_{il}^{-1} \left[\widetilde{\Phi}_{jli} \circ \eta_i(V_{i0})\right]^4 \quad (44)$$

where k_i's are positive constants. It can be checked that ϕ_i is a smooth $\mathcal{K}_\infty$ function by the construction (44) and $\phi_i(0) = 0$.

Then from (37) and (38) we have the dominance condition

$$\sum_{i=1}^{N} \alpha_i(|z_i|) \geq \sum_{i=1}^{N} D_i |z_i|^2 + \sum_{i=1}^{N} \sum_{j=1}^{N} \beta_{ij}(|z_j|) \quad (45)$$

where $D_i = k_i \lambda_{\min}(Q_i) - (n-\kappa) d_{1i}$. Choose $d_{1i} < k_i \lambda_{\min}(Q_i)/(n-\kappa)$ so that $D_i > 0$.

Then from (41) and (45) we obtain:

$$\dot{W} \leq \sum_{i=1}^{N} \left\{ -D_i |z_i|^2 - \sum_{l=1}^{n-\kappa} C_{il} \widetilde{\xi}_{il}^2 + \gamma_i |\omega_i|^2 \right\} \quad (46)$$

it follows $\dot{W} \leq \sum_{i=1}^{N} \left\{ -D_i |z_{i1}|^2 + \gamma_i |\omega_i|^2 \right\}$.

Taking the integral along time t, the $\mathcal{L}_2$ gain from ω to y of the closed-loop system is obtained as (6) with $\mu = \max_{1 \leq i \leq N}\{\gamma_i\} / \min_{1 \leq i \leq N}\{D_i\}$, and $\nu(z(0), \xi(0)) = W(z(0), \xi(0))$.

When $\omega_i = 0$, from (46),

$$\dot{W} \leq \sum_{i=1}^{N} \left\{ -D_i |z_i|^2 - \sum_{l=1}^{n-\kappa} C_{il} \widetilde{\xi}_{il}^2 \right\} \quad (47)$$

Since (from (7), (16), (35))

$$W(z, \xi) = \sum_{i=1}^{N} \left\{ \phi_i(V_{i0}(z_i)) + \sum_{l=1}^{n-\kappa} \widetilde{\xi}_{il}^2 \right\}$$

is a continuously differentiable, positive definite and radially unbounded function and its derivative is negative definite, we obtain GUAS of the closed-loop system when $\omega_i = 0$. □

4. AN ILLUSTRATIVE EXAMPLE

We consider the following interconnected system composed of two subsystems:

$$S_1 : \begin{cases} \dot{z}_{11} = z_{12} \\ \dot{z}_{12} = \xi_{11} + \sin(2t) z_{21} e^{z_{21}} + (1 + z_{11} z_{21}) \omega_1 \\ \dot{\xi}_{11} = u_1 + \xi_{11} z_{21} \omega_1 \\ y_1 = z_{11} \end{cases}$$

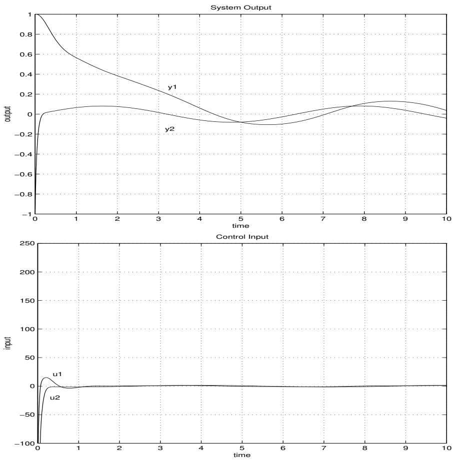

Fig. 1. Response of the closed-loop system

$$S_2 : \begin{cases} \dot{z}_{21} = \xi_{21} + z_{11} z_{12} \sin(\xi_{21}) + \omega_2 \\ \dot{\xi}_{21} = u_2 + \omega_2 \\ y_2 = z_{21} \end{cases}$$

It can be checked that system S is in form (1) and satisfies Assumptions 1 and 2. We choose the design parameters $\varepsilon_1 = 1; \varepsilon_2 = 1; Q_1 = 2I; Q_2 = 2$ to get

$$P_1 = \begin{bmatrix} 2.8284 & 1.0000 \\ 1.0000 & 1.4142 \end{bmatrix}, \quad P_2 = 1.$$

Then follow the design procedure presented above and choose $k_i = 8; \delta_{il} = \varrho_{il} = 0.1; \tau_i = 1; c_i = 0.1; d_{2i} = 2$ for $i = 1, 2; l = 1, 2$. The simulation results are shown in Figure 1 with the initial condition $[1, 0, 0, -1, 0]$. The disturbances used are $\omega_1 = \omega_2 = \sin(t)$.

5. CONCLUSIONS

The problem of decentralized H_∞ almost disturbance decoupling was solved for systems (1). A stepwise procedure which applied backstepping was developed and a decentralized controller was explicitly constructed. Our results extended the recent decentralized adaptive control work (Jain and Khorrami, 1997) which restricted time-invaraint interconnections to polynomial bounds. Simulations exhibited a good performance of a two-subsystem example.

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Appendix (Proof of the Claim): Consider the Lyapunov function as defined in (35). Differentiating it along the solutions of the $(z_i, \xi_{i1}, \dots, \xi_{il})$-subsystem, we have

$$\begin{aligned}\dot{W}_{il} \le& -\alpha_{i,l-1}(|z_i|) - \sum_{k=1}^{l-1} C_{i,l-1,k}\tilde{\xi}_{ik}^2 \\ &+ \sum_{j=1}^{N} \beta_{i,l-1,j}(|z_j|) + \gamma_{i,l-1}|\omega_i|^2 \\ &+2\tilde{\xi}_{il}\,[\xi_{i,l+1} + \Xi_{il}(z_i, \xi_{i1}, \dots, \xi_{il}) \\ &+\Gamma_{il}(t, 0, \xi_{i1}, \dots, \xi_{il}) + E_{il}(t, 0, \xi_{i1}, \dots, \xi_{il})\omega_i \\ &+F_{il}(t, z, \xi_{i1}, \dots, \xi_{il}) + G_{il}(t, z, \xi_{i1}, \dots, \xi_{il})\omega_i]\end{aligned} \tag{48}$$

where

$$\begin{aligned}\Xi_{il} =& \frac{1}{2}\frac{\partial W_{i,l-1}}{\partial \xi_{i,l-1}} - \frac{\partial \xi_{il}^*}{\partial z_i}(A_i z_i + B_i \xi_{i1}) \\ &-\sum_{k=1}^{l-1} \frac{\partial \xi_{il}^*}{\partial \xi_{ik}} \xi_{i,k+1} \ ,\end{aligned} \tag{49}$$

$$\Gamma_{il} = \sum_{k=1}^{l} \sigma_{ik}\gamma_{ik}(t, 0, \xi_{i1} \dots, \xi_{ik}) \ , \tag{50}$$

$$E_{il} = \sum_{k=0}^{l} \sigma_{ik} p_{ik}(t, 0, \xi_{i1}, \dots, \xi_{ik}) \ , \tag{51}$$

$$\begin{aligned}F_{il} =& \sum_{k=0}^{l} \sigma_{ik}\,[\gamma_{ik}(t, z, \xi_{i1} \dots, \xi_{ik}) \\ &-\gamma_{ik}(t, 0, \xi_{i1} \dots, \xi_{ik})] \ ,\end{aligned} \tag{52}$$

$$\begin{aligned}G_{il} =& \sum_{k=0}^{l} \sigma_{ik}\,[p_{ik}(t, z, \xi_{i1} \dots, \xi_{ik}) \\ &-p_{ik}(t, 0, \xi_{i1} \dots, \xi_{ik})]\end{aligned} \tag{53}$$

with σ_{i0} defined in (17), $\sigma_{ik} = -\frac{\partial \xi_{il}^*}{\partial \xi_{ik}}$ when $k = 1, \dots, l-1$ and $\sigma_{il} = 1$.

To handle time-varying term Γ_{il}, firstly we notice the mapping

$$\Theta : (z_i, \xi_{i1}, \xi_{i2}, \dots, \xi_{il}) \mapsto (z_i, \tilde{\xi}_{i1}, \tilde{\xi}_{i2}, \dots, \tilde{\xi}_{il})$$

is a global diffeomorphism since its Jacobian matrix is lower triangular with all diagonal components equal to the constant one. And it preserves the origin. Therefore, Θ^{-1} is smooth and also preserves the origin. So there exists a smooth function $\tilde{\psi}_{ik}$ such that

$$\begin{aligned}\psi_{ik}(|(\xi_{i1}, \dots, \xi_{ik})|) \le& \tilde{\psi}_{ik}(z_i, \tilde{\xi}_{i1}, \dots, \tilde{\xi}_{ik}) \\ &\cdot \left(|z_i| + \sum_{\iota=1}^{k} |\tilde{\xi}_{i\iota}|\right)\end{aligned} \tag{54}$$

Using the bounds from Assumption 1 and Assumption 2, there exists a smooth function $\Delta_{il}(\cdot)$ such that (with $d_{3i} > 0$)

$$\begin{aligned}&2\tilde{\xi}_{il}\Gamma_{il}(t, 0, \xi_{i1}, \dots, \xi_{il}) \\ &\le 2|\tilde{\xi}_{il}| \sum_{k=1}^{l} |\sigma_{ik}|\tilde{\psi}_{ik}(z_i, \tilde{\xi}_{i1}, \dots, \tilde{\xi}_{ik}) \left(|z_i| + \sum_{\iota=1}^{k} |\tilde{\xi}_{i\iota}|\right) \\ &\le \tilde{\xi}_{il}^2 \Delta_{il}(z_i, \tilde{\xi}_{i1}, \dots, \tilde{\xi}_{il}) + \left[d_{1i}|z_i|^2 + \sum_{k=1}^{l-1} d_{3i}|\tilde{\xi}_{ik}|^2\right]\end{aligned} \tag{55}$$

Similarly,

$$\begin{aligned}&2\tilde{\xi}_{il}E_{il}(t, 0, \xi_{i1}, \dots, \xi_{il})\omega_i \\ &\le \frac{1}{d_{2i}}|\tilde{\xi}_{il}|^2 \left[\sum_{k=0}^{l} |\sigma_{ik}|\Psi_{ik}\right]^2 + d_{2i}|\omega_i|^2 \ ,\end{aligned} \tag{56}$$

$$\begin{aligned}&2\tilde{\xi}_{il}F_{il}(t, z, \xi_{i1}, \dots, \xi_{il}) \\ &\le \tilde{\xi}_{il}^2 \sum_{k=0}^{l} \sigma_{ik}^2 \delta_{jk} a_{ikj}^2 + \sum_{k=0}^{l}\sum_{j=1}^{N} \delta_{jk}^{-1}\varphi_{ikj}^2(|z_j|),\end{aligned} \tag{57}$$

$$\begin{aligned}&2\tilde{\xi}_{il}G_{il}(t, z, \xi_{i1}, \dots, \xi_{il})\omega_i \\ &\le \frac{1}{4\tau_i^2}\tilde{\xi}_{il}^4 \sum_{k=1}^{l} \sigma_{ik}^4 \varrho_{jk} b_{ikj}^4 + \sum_{k=0}^{l}\sum_{j=1}^{N} \varrho_{jk}^{-1}\Phi_{ikj}^4(|z_j|) \\ &\quad +N\tau_i|\omega_i|^2\end{aligned} \tag{58}$$

So we choose virtual controller $\xi_{i,l+1} = \xi_{i,l+1}^*$ as

$$\begin{aligned}\xi_{i,l+1}^* = -\frac{1}{2}\Bigg\{& c_{il}\tilde{\xi}_{il} + 2\Xi_{il} + \tilde{\xi}_{il}\Delta_{il}(z_i, \tilde{\xi}_{i1}, \dots, \tilde{\xi}_{il}) \\ &+\frac{1}{d_{2i}}\tilde{\xi}_{il}\left[\sum_{k=0}^{l} |\sigma_{ik}|\Psi_{ik}\right]^2 + \tilde{\xi}_{il}\sum_{k=0}^{l}\sum_{j=1}^{N} \sigma_{ik}^2\delta_{jk}a_{ikj}^2 \\ &+\frac{1}{4\tau_i^2}\tilde{\xi}_{il}^3 \sum_{k=1}^{l}\sum_{j=1}^{N} \sigma_{ik}^4 \varrho_{jk} b_{ikj}^4 \Bigg\}\end{aligned} \tag{59}$$

Substituting (59) into (48), we get (30) which completes the proof of the Claim. □