EE 471: Transport Phenomena in Solid State Devices

Spring 2018

Lecture 3

Transport in Semiconductors

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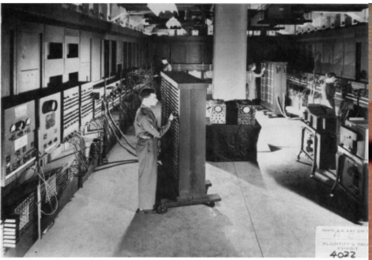
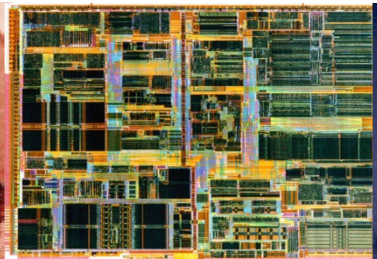
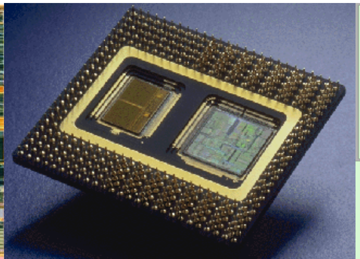
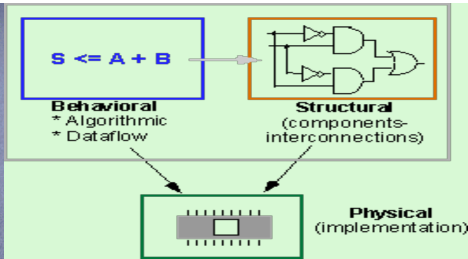
Adapted from Modern Semiconductor Devices for Integrated Circuits, Chenming Hu, 2010



Thermal Motion

- Carriers (conduction electrons, holes) have a finite kinetic energy which is a function of temperature
- For conduction electrons, kinetic energy = $(E - E_c)$

$$average\ electron\ kinetic\ energy = \frac{total\ kinetic\ energy}{number\ of\ electrons}$$

$$= \frac{\int f(E).D(E).(E - E_c)dE}{\int f(E).D(E)dE}$$

- Substituting and using Boltzmann approximation:

$$average\ kinetic\ energy = \frac{3}{2}.kT$$

  - Same result for holes
  - Statistical mechanics: $kT/2$ per degree of freedom

Thermal Velocity

$$\frac{1}{2} m_n . {v_{th}}^2 = \frac{3}{2} . kT$$

where m_n is effective electron mass, v_{th} *is thermal velocity*

$$v_{th} = \sqrt{\frac{3kT}{m_n}}$$

- At 300°K, and using $m_n = 0.26\ m_0$

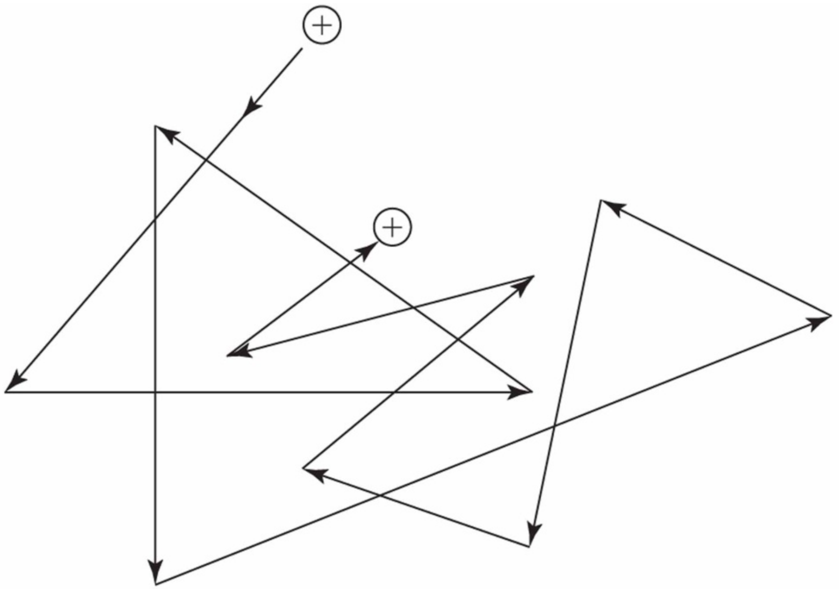
$$v_{th} = \sqrt{\frac{3 \times 1.38 \times 10^{-23} \mathrm{J/K} \times 300\mathrm{K}}{0.26 \times 9.1 \times 10^{-31} \mathrm{kg}}}$$

$$= 2.3 \times 10^5\ \mathrm{m/sec} = 2.3 \times 10^7 \mathrm{cm/sec}$$

≈ 670 times speed of sound in air

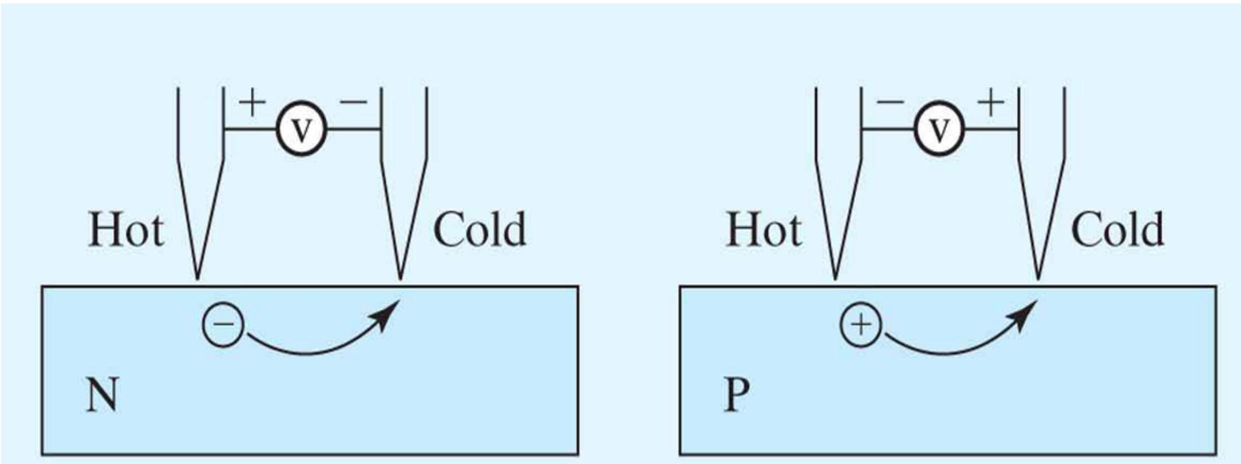
or 1/1000 speed of light

Scattering

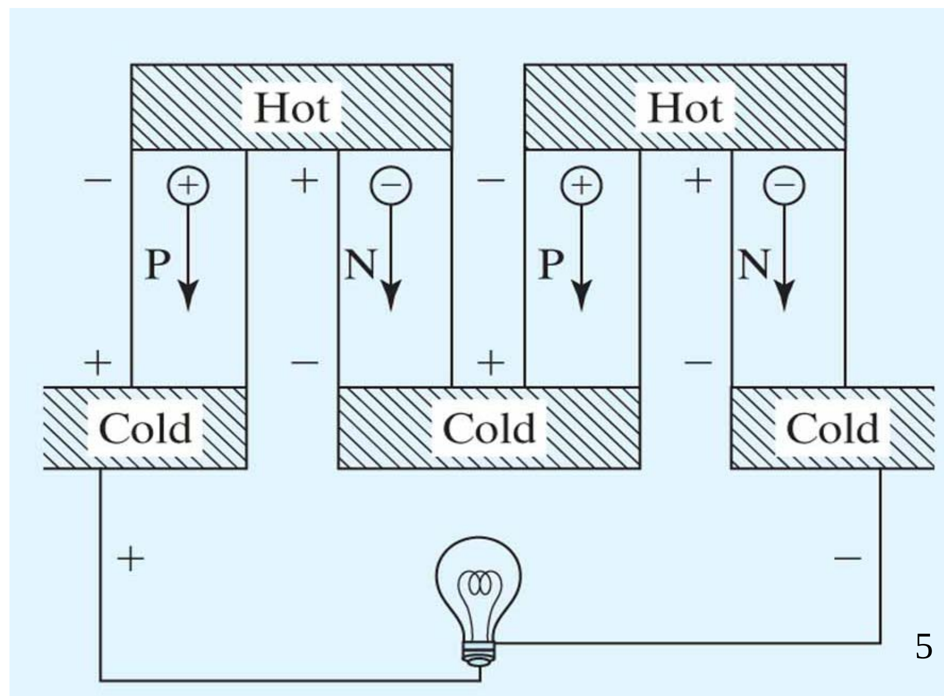


- Electrons and holes move in a zig-zag pattern due to collisions or scattering of atoms in the crystal
- At thermal equilibrium, net thermal velocity is zero
  - does not generate DC current, but does generate noise
- mean free time between collisions ≈ 0.1 ps at 300°K
- mean distance between collisions ≈ 25 nm
  - about 50x lattice constant

Non-equilibrium conditions



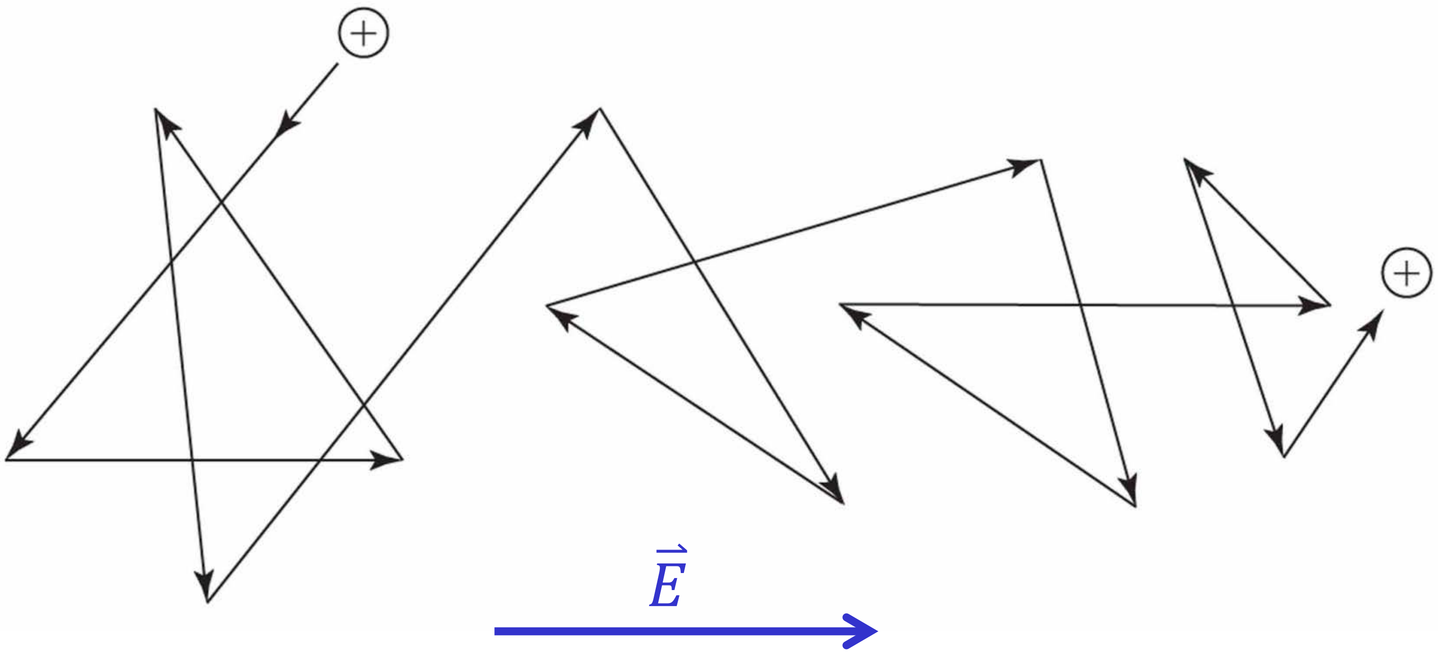
Hot-point probe: *distinguishes N and P-type semiconductor*



Thermo-electric generator : *converts heat to electric power*

Or a thermo-electric cooler: *solid-state refrigerator*

Drift



- Drift is the motion of charged carriers in presence of an electric field
- Drift is superimposed on thermal motion
- Drift velocity v_{drift} is average net velocity due to electric field

Drift Velocity

- Suppose we have a block of semiconductor that contains N_p holes under the influence of an electric field $\vec{E}$.
  - each hole is accelerated by electric field until it collides with the lattice
  - assume each hole loses its entire drift momentum at each collision

- Total drift momentum being added to all the holes by the electric field in time Δt is given by:

$$\Delta p_E = N_p.\vec{E}.q.\Delta t$$

- If τ_{mp} is the mean free time between collisions, then there will be:

$(N_p.\Delta t)/\tau_{mp}$ collisions in time Δt

- Total drift momentum lost to collisions is given by:

$$\Delta p_C = \frac{-N_p.\Delta t.(mean\ hole\ drift\ momentum)}{\tau_{mp}} = \frac{-N_p.\Delta t.m_p.v_{p,drift}}{\tau_{mp}}$$

where $v_{p,drift}$ is the mean hole drift velocity

Drift Velocity and Mobility

- In steady state, $\Delta p = (\Delta p_E + \Delta p_C)$=0

which means:

$$\frac{N_p . \Delta t . m_p . v_{p,drift}}{\tau_{mp}} = N_p . q . \vec{E} . \Delta t \quad \Longrightarrow \quad v_{p,drift} = \frac{q . \vec{E} . \tau_{mp}}{m_p}$$

$$v_{p,drift} = \mu_p . \vec{E}$$

$$\mu_p = \frac{q . \tau_{mp}}{m_p}$$

$$v_{n,drift} = -\mu_n . \vec{E}$$

$$\mu_n = \frac{q . \tau_{mn}}{m_n}$$

μ_p is the hole mobility

μ_n is the electron mobility

Electron & Hole Mobilities

$v = \mu . \vec{E}$: μ has the dimensions of $v / \vec{E}$

$$\frac{cm/s}{V/cm} \equiv \frac{cm^2}{V.s}$$

- Electron & hole mobilities, room temperature, lightly doped:

	Si	**Ge**	**GaAs**	**InAs**
μ_n (cm²/V·s)	1400	3900	8500	30000
μ_p (cm²/V·s)	470	1900	400	500

- Mobility describes ability of carrier to respond to $\vec{E}$
- Higher mobility leads to higher speed devices

Example: Drift Velocity

- Given μ_p = *470 cm²/V.s* for Si, what is the hole drift velocity at $\vec{E}$ = *10³ V/cm ?*

- What is the mean free path?

Effective Mass of Electron (Hole) ???

- We have used the term effective mass of electrons and holes (m_n, m_p) in both calculation of density of quantum states and in calculation of mobility.
- Effective mass has different value in these two formulae
  - because particle is moving through 3-D lattice and effective mass is single term that tries to capture effects of lattice in all dimensions
  - in density of states: $m_d = K_1.\left(m_x.m_y.m_z\right)^{1/3}$
  - in mobility: $m_b = K_2.\left(1/m_x + 1/m_y + 1/m_z\right)^{-1}$

	Density of States	Mobility
m_n/m_0	1.08	0.26
m_p/m_0	0.56	0.386

where $m_0 = 9.11 \times 10^{-31} kg$

Phonon Scattering

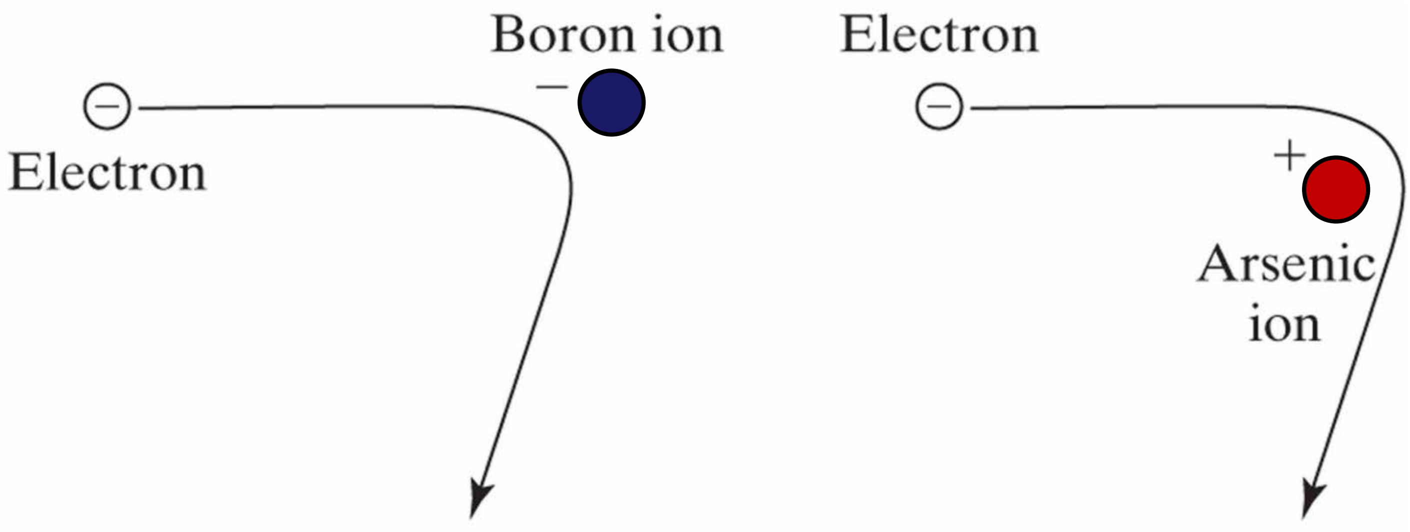
- Mobility depends on τ_{mn} and τ_{mp}
  - What determines mean time between scattering (collisions)?
- Electrons & holes are scattered by non-uniformities in crystal lattice. Two main causes:
  - Phonon scattering
  - Ionized impurity scattering
- Phonons are (quantum mechanical) particle representation of thermal vibration in the crystal lattice

$$\tau_{ph} \propto \frac{1}{phonon\ density \times carrier\ therm.velocity} \propto \frac{1}{T \times T^{1/2}}$$

- so $\mu_{ph} \propto \tau_{ph} \propto T^{-3/2}$
- Phonon-limited mobility decreases with temperature

Ionized Impurity Scattering

- Electrons and holes can be scattered by donor (positive) ion or acceptor (negative) ion:

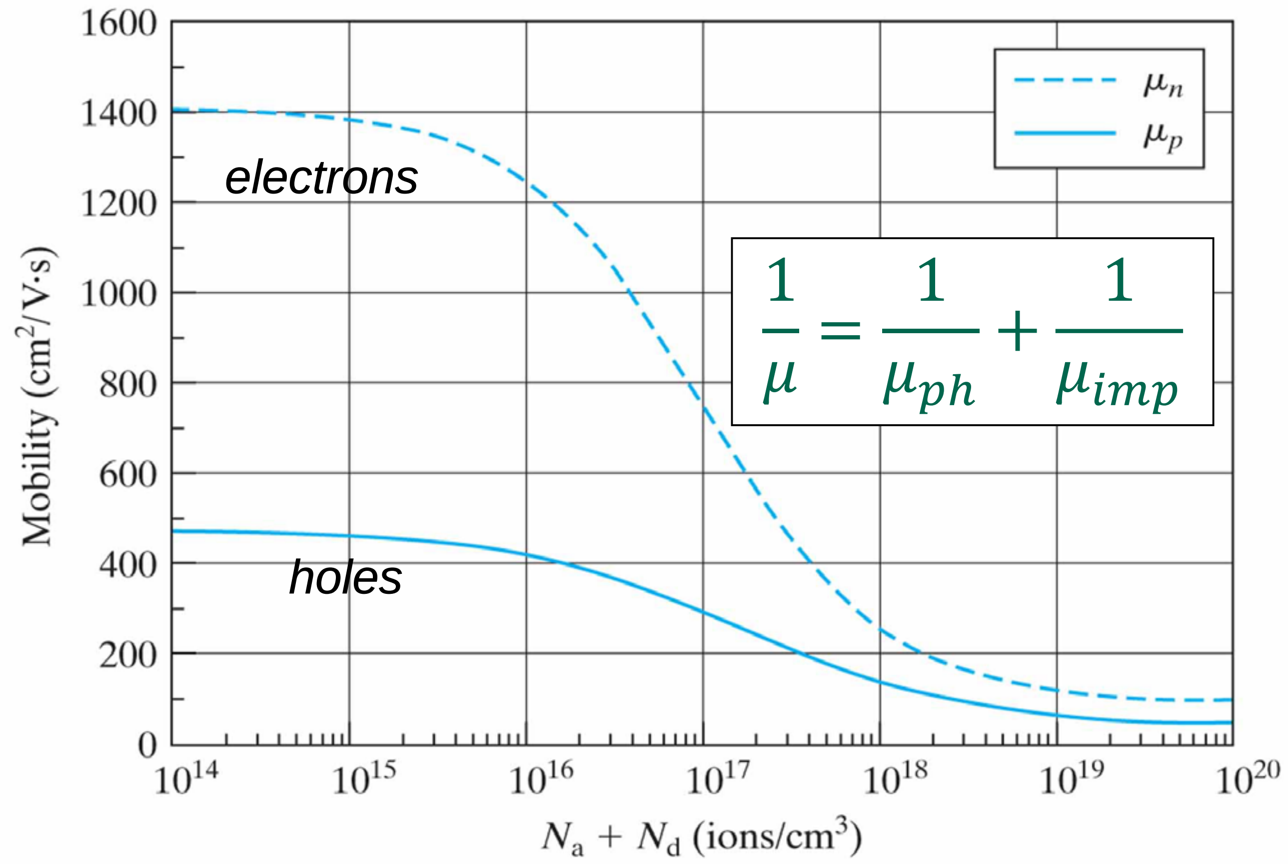


- Less change in direction if electron is travelling at higher speed

$$\mu_{imp} \propto \tau_{imp} \propto \frac{(carrier\ therm\ velocity)^3}{impurity\ density} \propto \frac{T^{3/2}}{N_a + N_d}$$

- Impurity-limited mobility increases with temperature

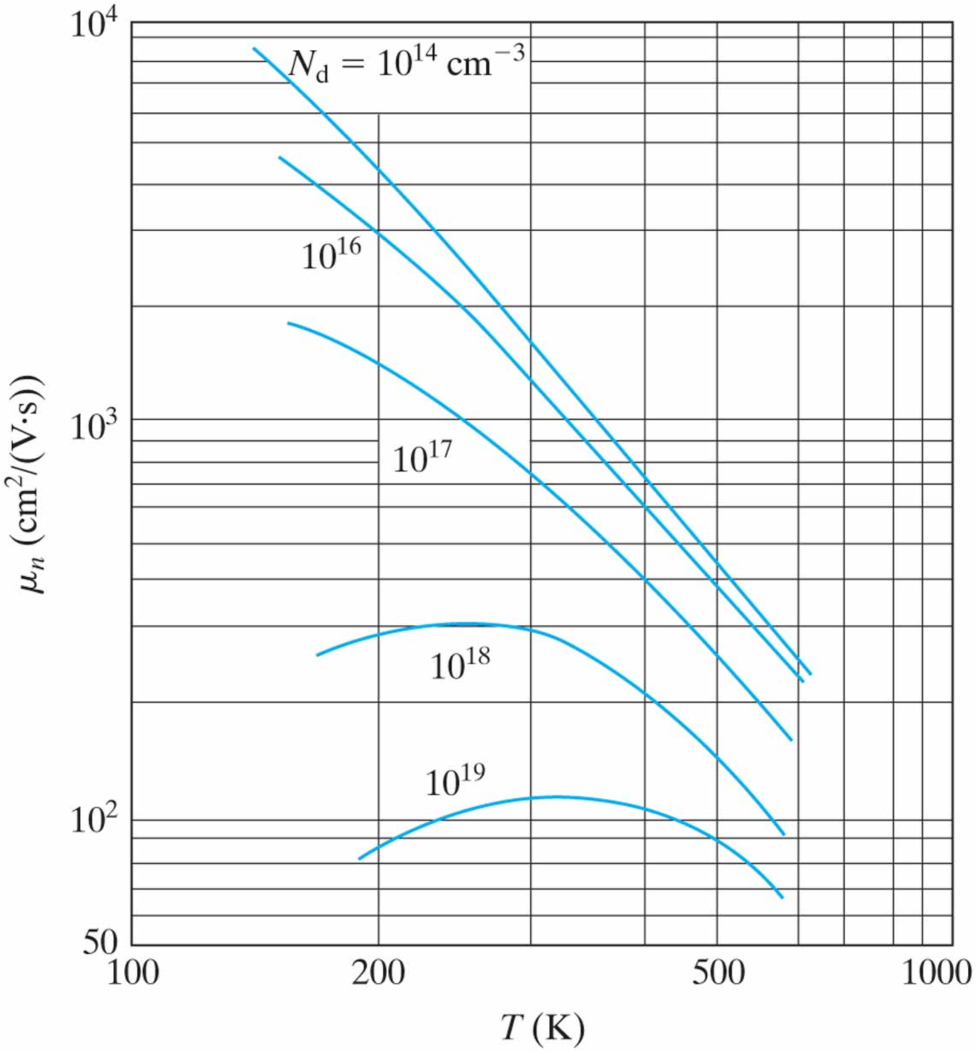
Electron & Hole Mobility

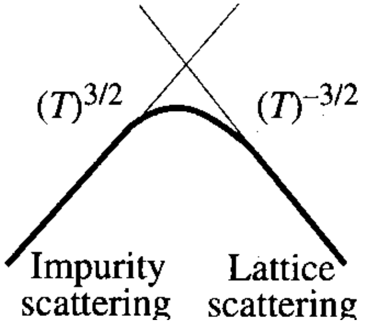


$$\frac{1}{\mu} = \frac{1}{\mu_{ph}} + \frac{1}{\mu_{imp}}$$

- Holes have about 1/3 mobility of electrons
- At low dopant concentration, mobility dominated by phonon scattering
- At high concentration, mobility further reduced by impurity scattering

Mobility vs. Temperature





- At high dopant concentrations and low temperature, mobility is dominated by impurity ion scattering

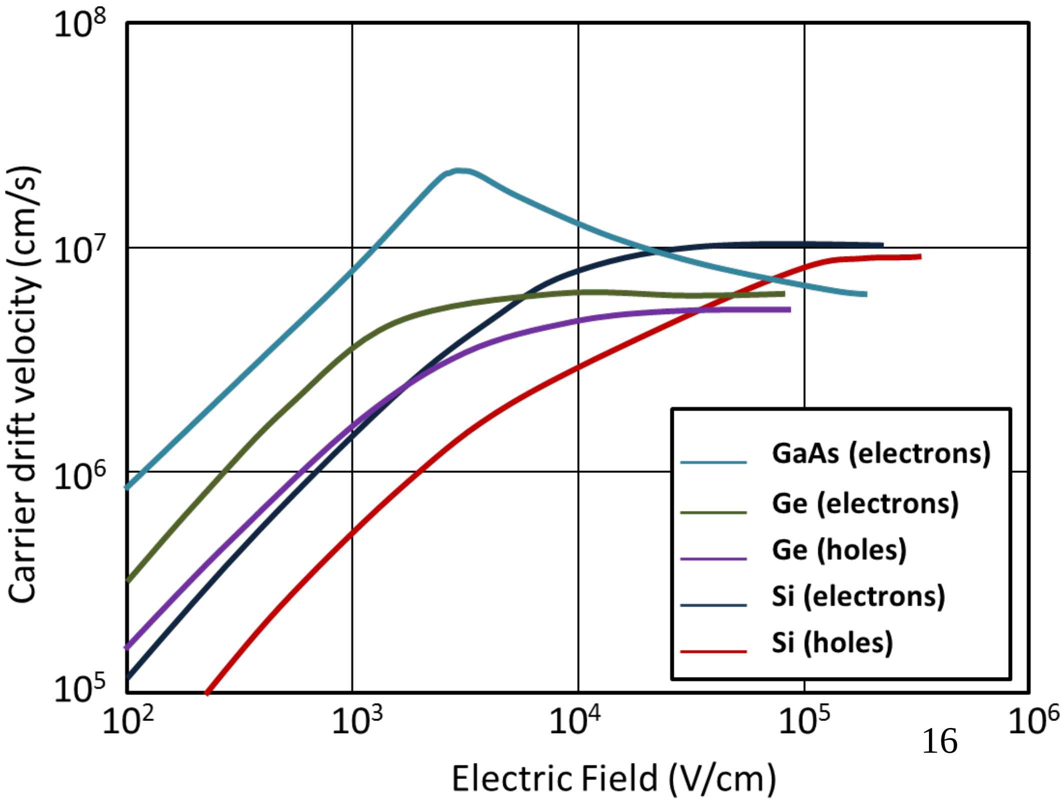
Velocity Saturation

- Under low electric fields, drift velocity << thermal velocity
- Under high field conditions, drift velocity adds appreciably to overall kinetic energy of electron (hole)
- This reduces mean free time between collisions which increases phonon induced scattering (which reduces μ)
- Drift velocity saturates:

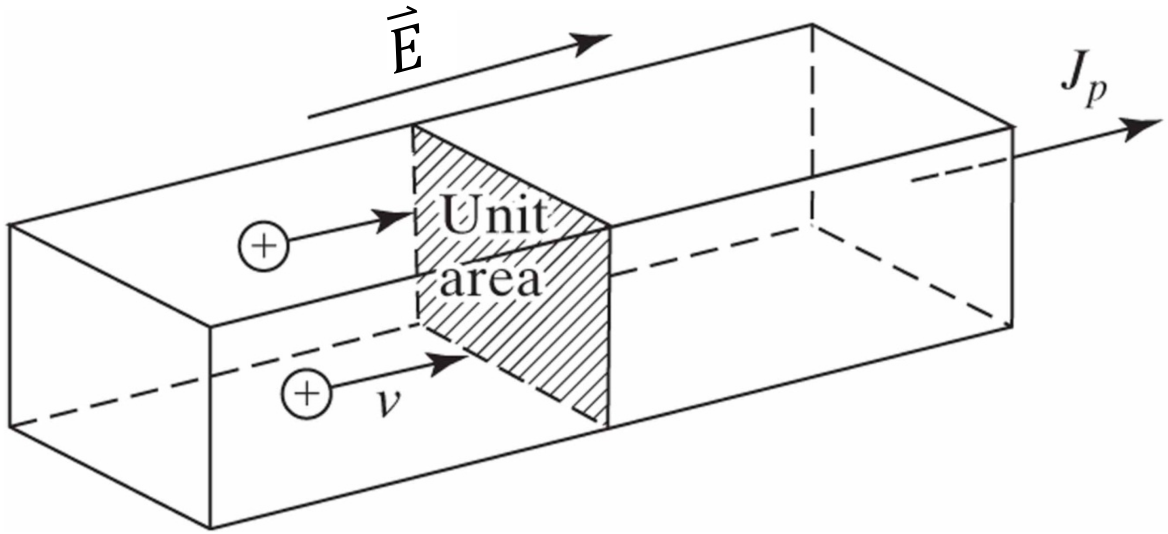
For Si at 300°K:

$v_{sat} \approx 10^7$ *cm/s*

at $\vec{E} \approx 10^5\ V/cm$



Carrier Velocity and Drift Current



- Current density J is charge per second crossing unit area plane normal to direction of current flow (units are A/cm^2)

$$J_{p,drift} = q.p.v_{p,drift}$$

- Example: if $p = 10^{15}/cm^{-3}$ and $v = 10^4 cm/s$:

$$J_{p,drift} = 1.6 \times 10^{-19} \times 10^{15} \times 10^4$$
$$= 1.6\ A/cm^2$$

Drift Current and Conductivity

$$J_{p,drift} = q.p.v_{p,drift} = q.p.\mu_p.\vec{E}$$

$$J_{n,drift} = -q.n.v_{n,drift} = q.n.\mu_n.\vec{E}$$

$$J_{drift} = (J_{n,drift} + J_{p,drift}) = (q.n.\mu_n + q.p.\mu_p).\vec{E}$$

- Define conductivity σ as: $J_{drift} = \sigma.\vec{E}$
  - Units of σ are $(\text{ohm.cm})^{-1}$
- then $\boxed{\sigma = \frac{1}{\rho} = q.n.\mu_n + q.p.\mu_p}$ (ρ is resistivity)

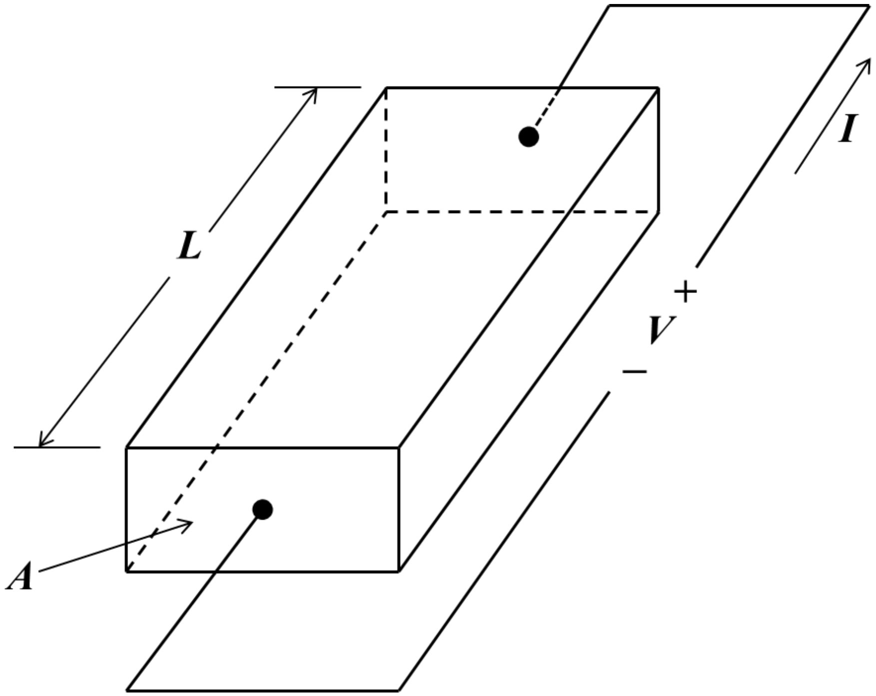
Ohms Law

- If we have a bar of semiconductor:

$$J_{drift} = \frac{I}{A}$$

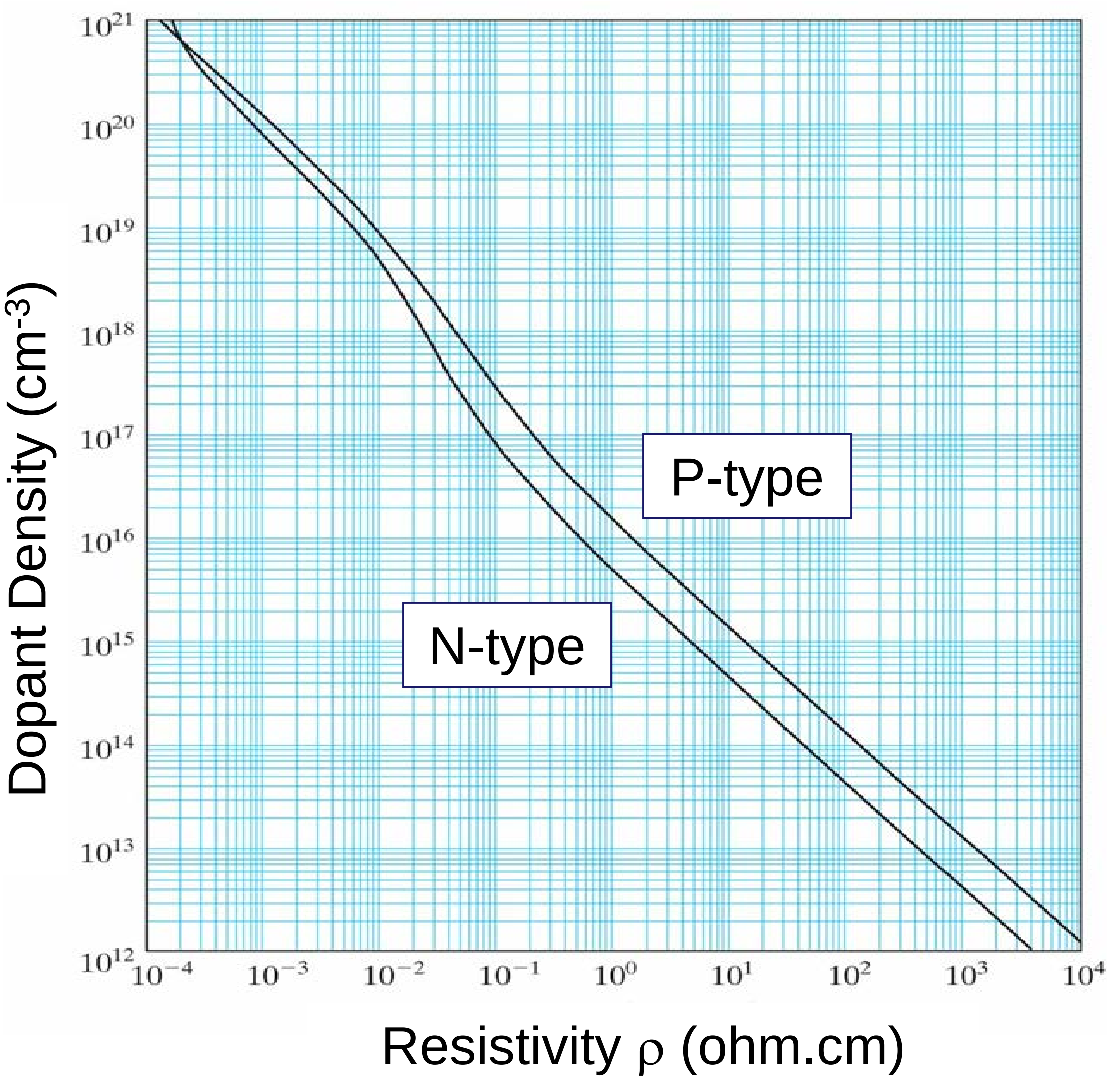
$$\vec{E} = \frac{V}{L}$$

$$\Rightarrow \quad \frac{I}{A} = \sigma\left(\frac{V}{L}\right)$$



i.e. $V = I.R$ where $R = \frac{\rho.L}{A}$

Resistivity vs. Dopant Density



$$\rho = \frac{1}{\sigma}$$

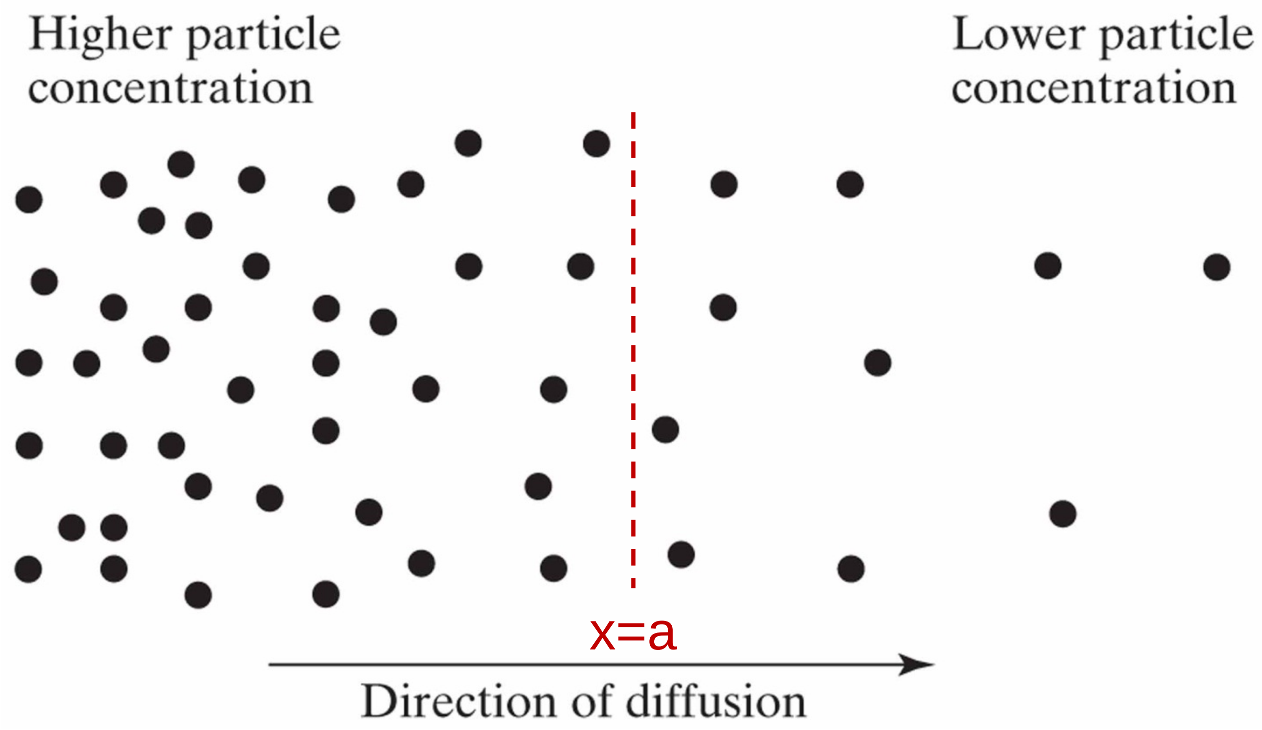
Example: Silicon Resistivity

a) What is the room temperature resistivity ρ of silicon doped with 10^{17} cm^{-3} of arsenic?

b) What is the resistance of a piece of this material that is $1\mu m$ long and $0.2\ \mu m^2$ in area?

c) By what factor will resistance change as we go from 27°C to 127°C?

Diffusion



- All particles are in constant thermal motion
- Because $(\text{density})_{x<a}$ is greater than $(\text{density})_{x>a}$, more particles cross x=a from left-to-right than from right-to-left
- Net particle flow from high to low concentration
- If particles are charged (carriers) ⇨ net current

Diffusion Current

- Rate of particle flow proportional to concentration gradient

$$flow\ per\ \text{unit area} = -\text{D}.\frac{d(concentration)}{dx}$$

  - where D is the diffusion constant

$$flow\ per\ \text{unit area} = concentration\ \times velocity$$

$$v_{n,diff} = \frac{-1}{n}.D_n.\frac{dn}{dx}$$

  - leads to a diffusion current density: $J_{n,diff} = -n.q.v_{n.diff}$

$$J_{n,diff} = q.D_n.\frac{dn}{dx}$$

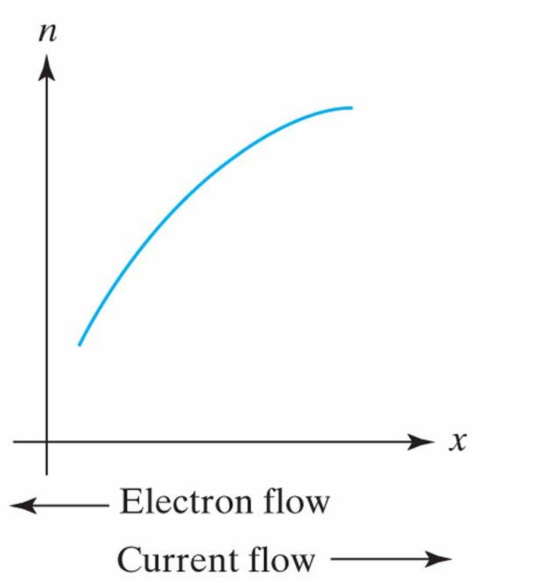
$$J_{p,diff} = -q.D_p.\frac{dp}{dx}$$

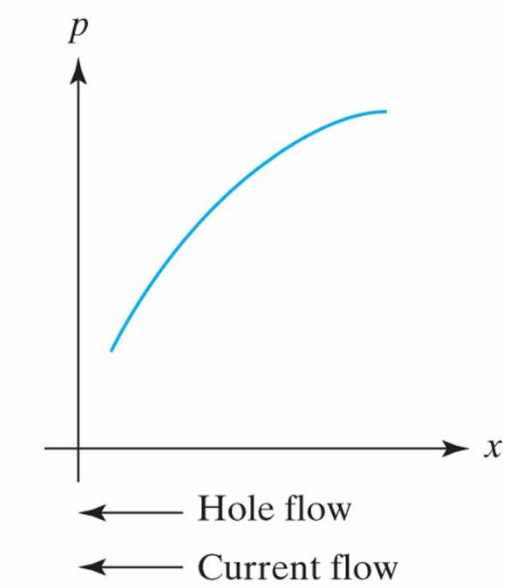
Diffusion Current

$$J_{n,diff} = q.D_n.\frac{dn}{dx}$$

$$J_{p,diff} = -q.D_p.\frac{dp}{dx}$$

- Units of Diffusion Constant (D_n, D_p) are cm^2/s





Example: Diffusion Current

- Assume that in a sample of N-type silicon the electron concentration varies linearly from 1×10^{18} to 7×10^{17} over a distance of $0.1 mm$. Calculate the diffusion current density if $D_n = 7.8\, cm^2/s$.

Total Current in Semiconductor

$$J = J_n + J_p$$

$$J_n = J_{n,drift} + J_{n,diff} = q.n.\mu_n.\vec{E} + q.D_n.\frac{dn}{dx}$$

$$J_p = J_{p,drift} + J_{p,diff} = q.p.\mu_p.\vec{E} - q.D_p.\frac{dp}{dx}$$

- Four terms in total
- Frequently only need to consider one term at any one time at any one point in semiconductor

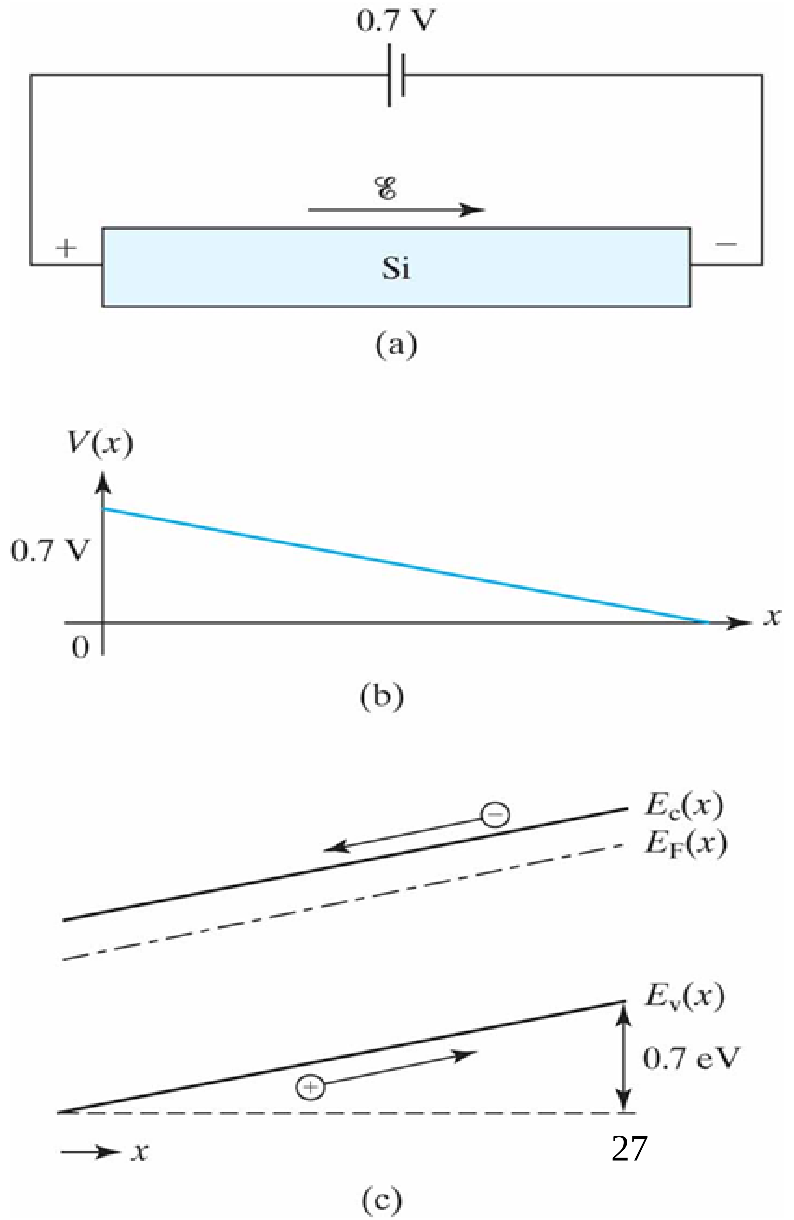
Applied Voltage and Energy Band Diagrams

- When a voltage is applied to a semiconductor, it tilts the band diagram
- Positive voltage raises potential of holes, decreases potential of electrons
  - lowers energy bands
  - E_c and E_v always separated by E_g

$$E_c(x) = constant - q.V(x)$$

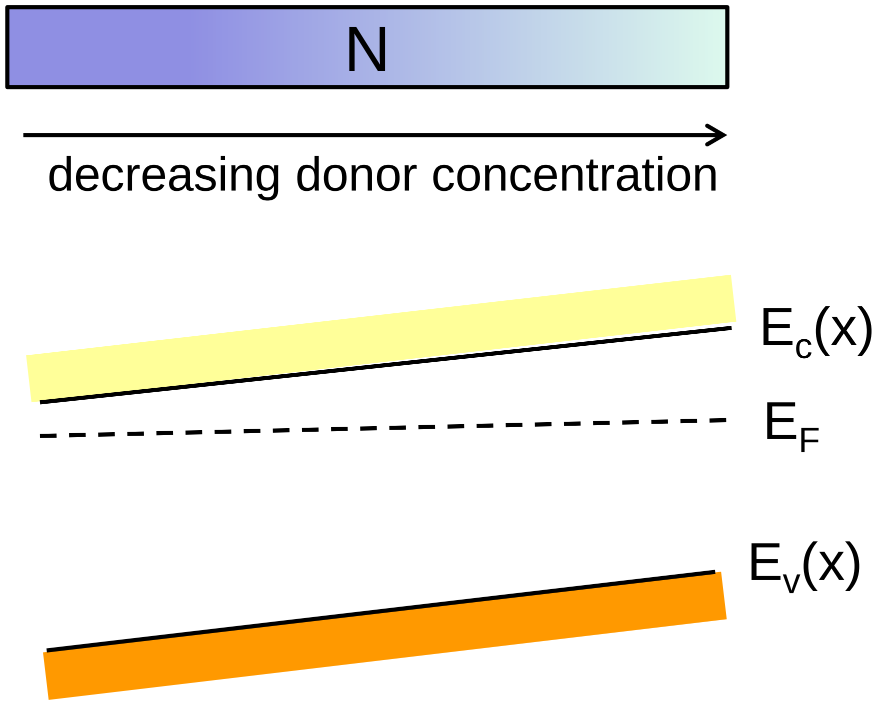
- *Remember: Ec and Ev move in <u>opposite</u> direction to applied V*

$$\vec{E}(x) = -\frac{dV}{dx} = \frac{1}{q}\frac{dE_c}{dx} = \frac{1}{q}\frac{dE_v}{dx}$$



Graded Impurity Distribution

- Consider a bar of N-type silicon <u>in thermal equilibrium</u>, more heavily doped on left side:



- Electrons will diffuse from left to right (diffusion current right to left)
- This moves negative charge (electrons) to RHS
- Creates electric field from left to right which induces drift current that draws electrons back to LHS
- Electrons diffuse until drift current balances diffusion current

- Note that band diagram is consistent with:
  - difference in carrier density (as seen in E_c-E_F)
  - presence of electric field $\vec{E} = \frac{1}{q}\frac{dE_c}{dx}$

Einstein Relationship

- If bar is in thermal equilibrium:

$$J_n = 0 = q.n.\mu_n.\vec{E} + q.D_n.\frac{dn}{dx}$$

- remember $\quad n = N_c.e^{-(E_c - E_F)/kT}$

$$\frac{dn}{dx} = \frac{-N_c}{kT} e^{-(E_c - E_F/kT)}.\frac{dE_c}{dx}$$

$$= \frac{-n}{kT}.\frac{dE_c}{dx} \quad = \frac{-n}{kT}.q.\vec{E}$$

- so $\quad 0 = q.n.\mu_n.\vec{E} - q.D_n.\frac{q.n}{kT}.\vec{E}$

$$D_n = \frac{kT}{q}.\mu_n$$

$$D_p = \frac{kT}{q}.\mu_p$$

Example: Diffusion Constant

- A piece of silicon is doped with $3 \times 10^{15} cm^{-3}$ of donors and $7 \times 10^{15} cm^{-3}$ of acceptors. What are the electron and hole diffusion constants at 300°K?

Example: Potential due to density gradient

- Consider a n-type semiconductor at T=300°K in thermal equilibrium. Assume that the donor concentration varies as $N_d(x) = N_{d0}.e^{-x/L}$ over the range $0 \leq x \leq 5L$ where $N_{d0} = 10^{16} cm^{-3}$ and $L = 10\mu m$.
  a) Determine the electric field as a function of x for $0 \leq x \leq L$.
  b) Calculate the potential difference between x=0 and x= 25 μm.

Example: Electron collision frequency

- An electron is moving in a piece of very lightly doped silicon at room temperature under an applied field such that its drift velocity is one-tenth of its thermal velocity. Calculate the average number of collisions it will experience in traversing by drift a region 1μm long.