Lecture 4

Generation and Recombination

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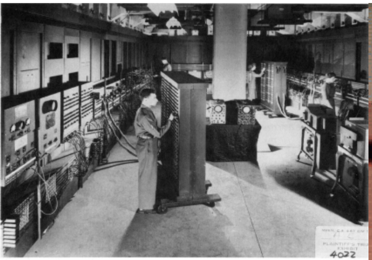
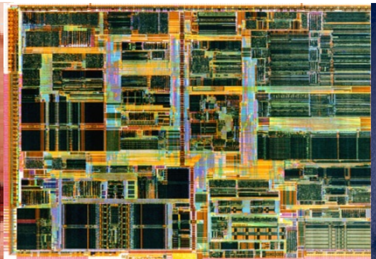
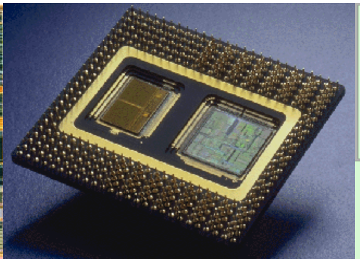
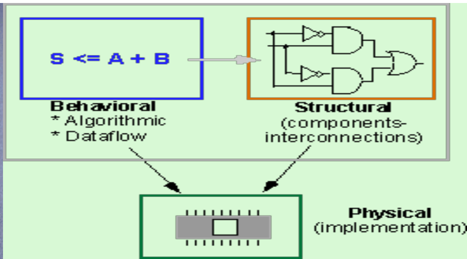
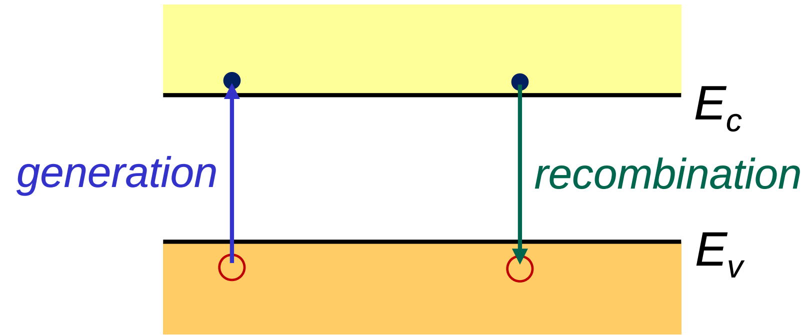
Adapted from Modern Semiconductor Devices for Integrated Circuits, Chenming Hu, 2010



Semiconductor in Equilibrium



- Under thermal equilibrium:
  $$n.p = {n_i}^2$$
- Electrons are continually being thermally excited from valence band to the conduction band
  - leaves behind a hole
  - known as electron-hole generation
  - occurs at rate $G\ cm^{-3}.s^{-1}$
- At same time, electrons are continually "falling" into holes
  - annihilates conduction electron and hole
  - known as electron-hole recombination
  - occurs at rate $R\ cm^{-3}.s^{-1}$
- Under thermal equilibrium: $G = R$

Excess Carriers

- Denote n_0, p_0 as the equilibrium carrier concentrations
  - then we can say $n_0 . p_0 = n_i{}^2$ at all times
- When not at equilibrium, n & p can be different from n_0 & p_0
  - for example, when light shines on semiconductor
  - or under external electric field
- Define excess carrier concentrations n' and p':

$$n \equiv n_0 + n'$$

$$p \equiv p_0 + p'$$

- Charge neutrality requires:

$$n' = p'$$

Recombination (Carrier) Lifetime

- At thermal equilibrium $R = R_0 = G = G_0$
- Suppose we generate excess carriers by shining light on a semiconductor
- Now $G = G_0 + G'$ and $n' = p' > 0$
- As concentration of excess carriers increases, recombination rate $R = R_0 + R'$ increases until $R' = G'$
- Net recombination rate R' is proportional to $n'(= p')$

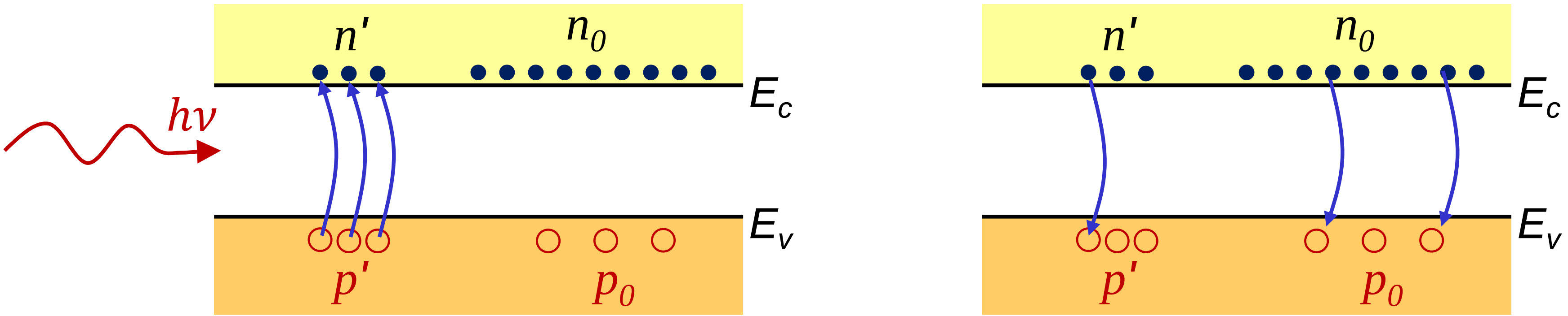
$$R' cm^{-3} s^{-1} \propto n' cm^{-3}$$

- Proportionality constant has dimensions of 1/time

$$R' = \frac{n'}{\tau}$$

- where τ is the recombination or carrier lifetime

Recombination Lifetime (cont.)

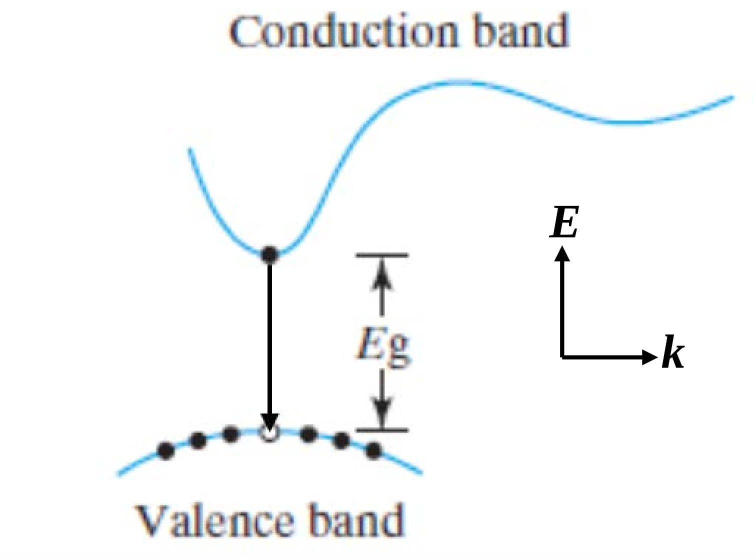


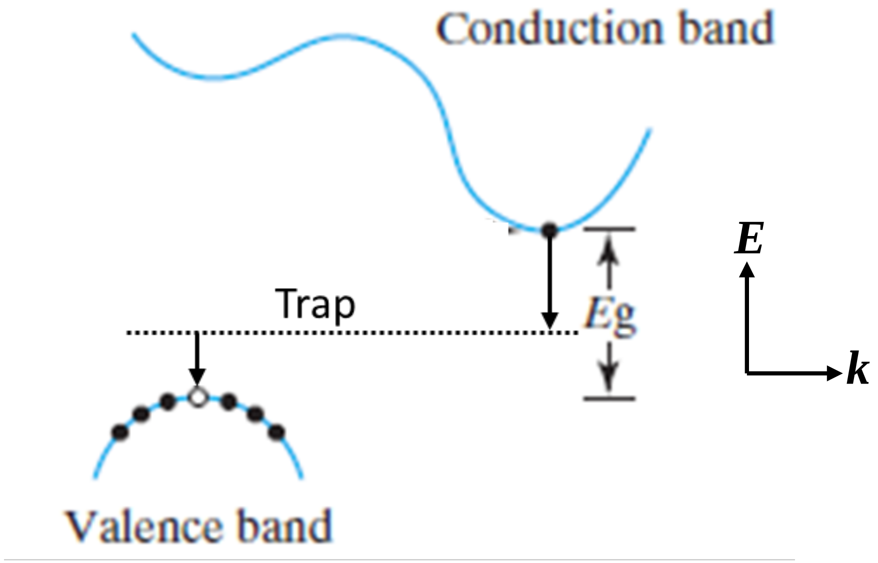
- Suppose we now turn light off (at time t=0)
- Recombination causes n' to decay (over time) to zero at which point thermal equilibrium is restored

$$\frac{dn'}{dt} = \frac{dp'}{dt} = -R' = \frac{-n'}{\tau} = \frac{-p'}{\tau}$$

- yields: $$n'(t) = n'(0)e^{-t/\tau}$$
- Carrier lifetime τ is time constant of excess carrier decay

Direct & Indirect Band Gaps

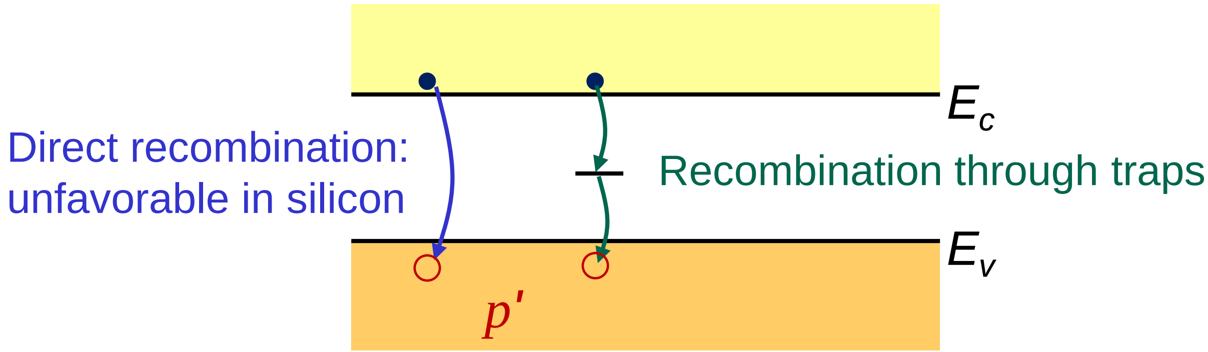




- **Direct Band Gap**
- *Example: GaAs*
- Direct recombination is possible since momentum is conserved
- Photon may be emitted as energy is released by electron

- **Indirect Band Gap**
- *Example: Si, Ge*
- Direct recombination is rare since momentum is not conserved
- Recombination is two-step process via trap

Direct & Indirect Band Gaps



- Trace metal impurities (Ag, Pt) have energy states deep in the band gap – these are called deep traps
- Conduction electrons can fall into trap and then subsequently fall from trap to hole in valence band
  - Interaction with crystal lattice allows momentum transfer
  - also called recombination centers
- Carrier lifetime in Si can range from *1ns* to *1ms* depending on level and type of contaminants
  - Carrier lifetime is used to test purity of a semiconductor
  - Indicator of leakage current

Generation & Recombination

- At thermal equilibrium:

$$n.p = {n_i}^2$$

$$n' = p' = 0$$

$$\frac{dn'}{dt} = \frac{-n'}{\tau} = 0$$: no net generation or recombination

- Whenever $n' = p' > 0$:

$$n.p > {n_i}^2$$

$$\frac{dn'}{dt} = \frac{-n'}{\tau} < 0$$: net thermal recombination

- Whenever $n' = p' < 0$:

$$n.p < {n_i}^2$$

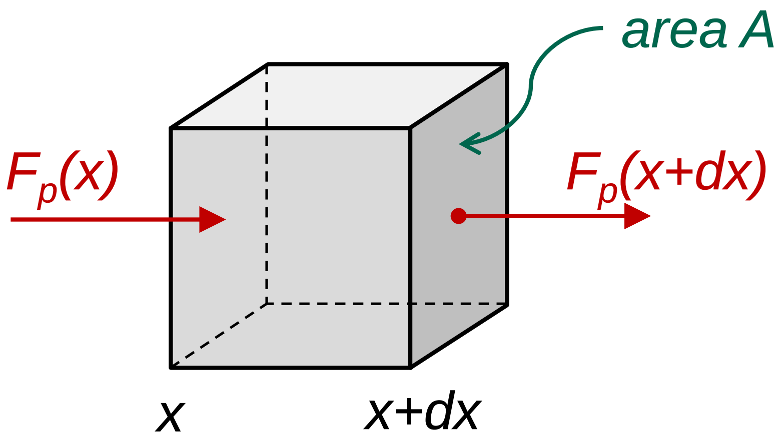
$$\frac{dn'}{dt} = \frac{-n'}{\tau} > 0$$: net thermal generation

Example: Photoconductors

- A bar of Si is doped with boron at 10^{15} cm^{-3}. It is exposed to light such that electron-hole pairs are generated throughout the volume of the bar at the rate of 10^{19}/cm^3.s. The recombination lifetime is 10μs. What are (a) p_0 , (b) n_0 , (c) n', (d) p', (e) p , (f) n and (g) the n.p product (h) n'(t) for t>0 if light is suddenly switched off at t=0?

Continuity Equations

- Relates carrier flux (current) to generation-recombination (G-R)



$F_p(x)$ is flux of holes per unit area per second

net increase in holes = increase due to flux + increase due to G-R

$$\frac{\partial p}{\partial t}.A.dx = \left[F_p(x) - F_p(x+dx)\right].A - \frac{p'}{\tau}.A.dx$$

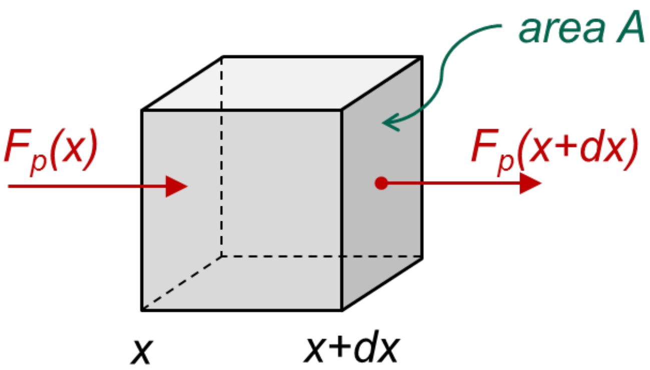
$$\Rightarrow \quad \frac{\partial p}{\partial t} = \frac{\left[F_p(x) - F_p(x+dx)\right]}{dx} - \frac{p'}{\tau}$$

$$\Rightarrow \quad \frac{\partial p}{\partial t} = -\frac{\partial F_p}{\partial x} - \frac{p'}{\tau}$$

Continuity Equations (cont.)

- Substituting $J_p(x) = q.F_p(x)$

$$\frac{1}{q}.\frac{\partial J_p}{\partial x} + \frac{p'}{\tau} = -\frac{\partial p}{\partial t}$$



- Similarly $-\frac{1}{q}.\frac{\partial J_n}{\partial x} + \frac{n'}{\tau} = -\frac{\partial n}{\partial t}$

- Since n_0 and p_0 do not change with time, $\frac{\partial p}{\partial t} = \frac{\partial p'}{\partial t}$ and $\frac{\partial n}{\partial t} = \frac{\partial n'}{\partial t}$:

$$-\frac{1}{q}.\frac{\partial J_n}{\partial x} + \frac{n'}{\tau_n} = -\frac{\partial n'}{\partial t}$$

$$\frac{1}{q}.\frac{\partial J_p}{\partial x} + \frac{p'}{\tau_p} = -\frac{\partial p'}{\partial t}$$

Diffusion Equations

- Substituting transport equation: $J_p = q.p.\mu_p.\vec{E} - q.D_p.\frac{dp}{dx}$

- into continuity equation: $\frac{1}{q}.\frac{\partial J_p}{\partial x} + \frac{p'}{\tau_p} = -\frac{\partial p'}{\partial t}$

- gives $\mu_p.\frac{\partial(p\vec{E})}{\partial x} - D_p.\frac{\partial^2 p}{\partial x^2} + \frac{p'}{\tau_p} = -\frac{\partial p'}{\partial t}$

- Expanding $\frac{\partial(p\vec{E})}{\partial x} = p.\frac{\partial \vec{E}}{\partial x} + \vec{E}.\frac{\partial p}{\partial x}$

$$D_p.\frac{\partial^2 p}{\partial x^2} - \mu_p\left(p.\ \frac{\partial \vec{E}}{\partial x} + \vec{E}.\frac{\partial p}{\partial x}\right) - \frac{p'}{\tau_p} = \frac{\partial p'}{\partial t}$$

$$D_n.\frac{\partial^2 n}{\partial x^2} + \mu_n\left(n.\ \frac{\partial \vec{E}}{\partial x} + \vec{E}.\frac{\partial n}{\partial x}\right) - \frac{n'}{\tau_p} = \frac{\partial n'}{\partial t}$$

- These are *time dependent diffusion equations*

Diffusion Equations: Special Cases

- If doping is uniform $\frac{\partial p}{\partial x} = \frac{\partial p'}{\partial x}$ which gives:

$$D_p.\frac{\partial^2 p'}{\partial x^2} - \mu_p\left(p.\ \frac{\partial \vec{E}}{\partial x} + \vec{E}.\frac{\partial p'}{\partial x}\right) - \frac{p'}{\tau_p} = \frac{\partial p'}{\partial t}$$

- Under steady state conditions, $\frac{\partial p'}{\partial t} = 0$ which gives:

$$D_p.\frac{d^2 p'}{dx^2} - \mu_p\left(p.\ \frac{d\vec{E}}{dx} + \vec{E}.\frac{dp'}{dx}\right) - \frac{p'}{\tau_p} = 0$$

- If E-field is negligible (minority carriers in neutral region):

$$\frac{d^2 p'}{dx^2} = \frac{p'}{D_p.\tau_p} = \frac{p'}{{L_p}^2}$$

$$\frac{d^2 n'}{dx^2} = \frac{n'}{D_n.\tau_n} = \frac{n'}{{L_n}^2}$$

$$L_p \equiv \sqrt{D_p.\tau_p}$$

$$L_n \equiv \sqrt{D_n.\tau_n}$$

- L_n and L_p are electron and hole diffusion lengths
  - can range from few μm to hundreds of μm depending on τ

Quasi-equilibrium and Quasi-Fermi Levels

- Whenever $n' = p' \neq 0, \quad n.p \neq {n_i}^2$
- We have these very useful equilibrium relationships:

$$n = N_c.e^{-(E_c - E_F)/kT}$$

$$p = N_v.e^{-(E_F - E_v)/kT}$$

- But these equations imply $n.p = {n_i}^2$
- For non-equilibrium, introduce two quasi-Fermi levels

$$n = N_c.e^{-(E_c - E_{Fn})/kT}$$

$$p = N_v.e^{-(E_{Fp} - E_v)/kT}$$

- At equilibrium, $E_{Fp} = E_{Fn} = E_F$, otherwise $E_{Fp} \neq E_{Fn}$
- Even when electrons and holes are not in equilibrium, *within each group* the carriers can be in equilibrium
  - electrons and holes loosely coupled via G-R (~ 1μs)
  - electrons (or holes) tightly coupled via scattering (0.1ps)

Example: Quasi-Fermi Levels & Low-Level Injection

- Consider a Si sample with:

$N_d = 10^{17} cm^{-3}$ and $n' = p' = 0$ *i.e. thermal equilibrium*

$$n_0 = N_d = 10^{17} cm^{-3} = N_c e^{-(E_c - E_F)/kT}$$

$$E_c - E_F = kT.\, ln\left(\frac{N_c}{10^{17} cm^{-3}}\right) = 26\, meV.\, ln\left(\frac{2.8 \times 10^{19} cm^{-3}}{10^{17} cm^{-3}}\right) = 0.15\, eV$$

so E_F is below E_c by $0.15\, eV$

- Now suppose we have $n' = p' = 10^{15} cm^{-3}$

No longer thermal equilibrium. But note that n' and p' are much less than the majority carrier concentration. This is called low-level injection.

Example: Quasi-Fermi Levels (cont.)

$$n = n_0 + n' = 1.01 \times 10^{17} cm^{-3} = N_c e^{-(E_c - E_{Fn})/kT}$$

$$E_c - E_{Fn} = 26\, meV.\, ln\left(\frac{2.8 \times 10^{19} cm^{-3}}{1.01 \times 10^{17} cm^{-3}}\right) \approx 0.15\, eV$$

so E_{Fn} is essentially unchanged from E_F because $n \approx n_0$

$$p = p_0 + p' = \frac{{n_i}^2}{N_d} + p' = 10^3 cm^{-3} + 10^{15} cm^{-3} \approx 10^{15} cm^{-3}$$

$$E_{Fp} - E_v = kT.\, ln\left(\frac{N_v}{10^{15} cm^{-3}}\right)$$

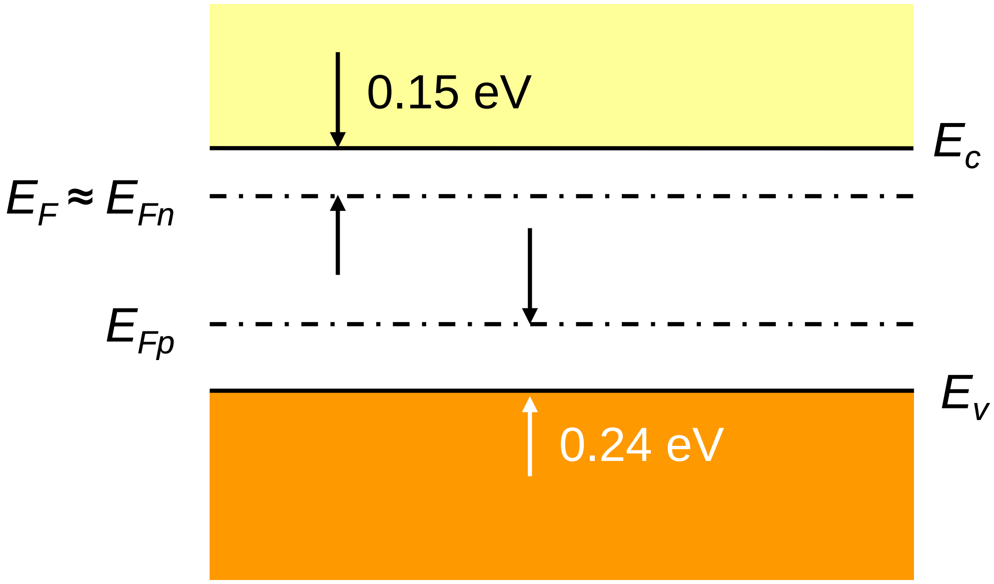
$$= 26\, meV.\, ln\left(\frac{1.04 \times 10^{19} cm^{-3}}{10^{15} cm^{-3}}\right) = 0.24\, eV$$

so E_{Fp} has shifted dramatically - is now above E_v by $0.24\, eV$

Example: Quasi-Fermi Levels (cont.)

$$E_c - E_{Fn} = 0.15\ eV$$

$$E_{Fp} - E_v = 0.24\ eV$$



Example: Diffusion Equation

- Consider a P-type semiconductor with $N_A = 10^{16}\ cm^{-3}$ at room temperature that is homogeneous and infinite in extent. Assume a zero applied electric field. For a one dimensional crystal, assume that excess carriers are being generated at $x = 0$ only (as shown in the Figure) and that $n'(0) = 10^{15}\ cm^{-3}$. If $\tau_n = 5 \times 10^{-7} s$ and $D_n = 25\ cm^2/s$,
  a) calculate the minority carrier diffusion length
  b) calculate the steady-state concentration of electrons and holes as a function of x