

EE 471: Transport Phenomena in Solid State Devices

Spring 2018

Lecture 8 MOS Transistor

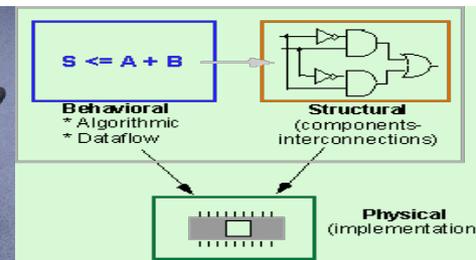
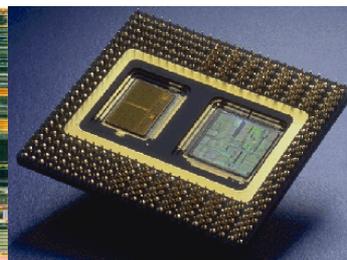
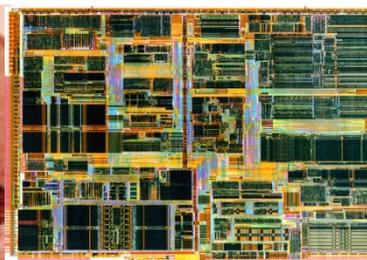
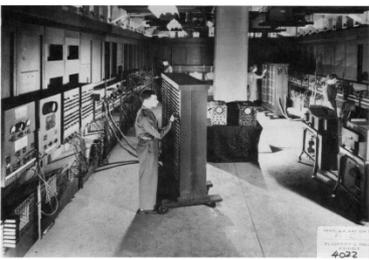
Bryan Ackland

Department of Electrical and Computer Engineering

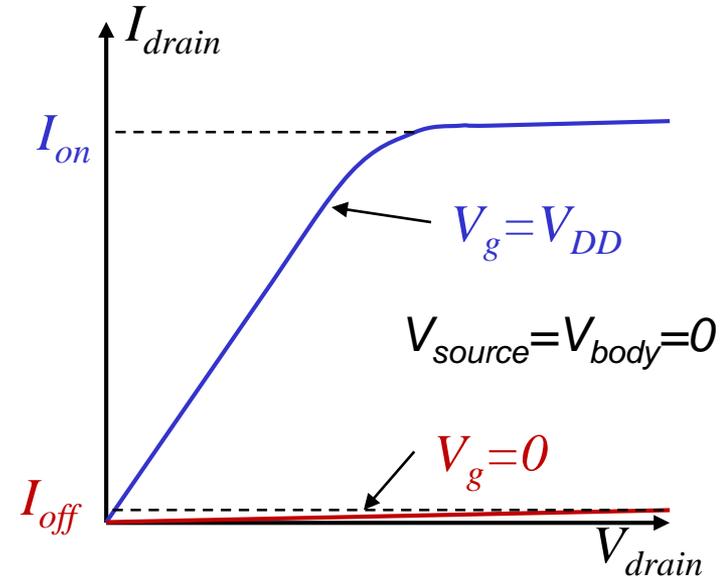
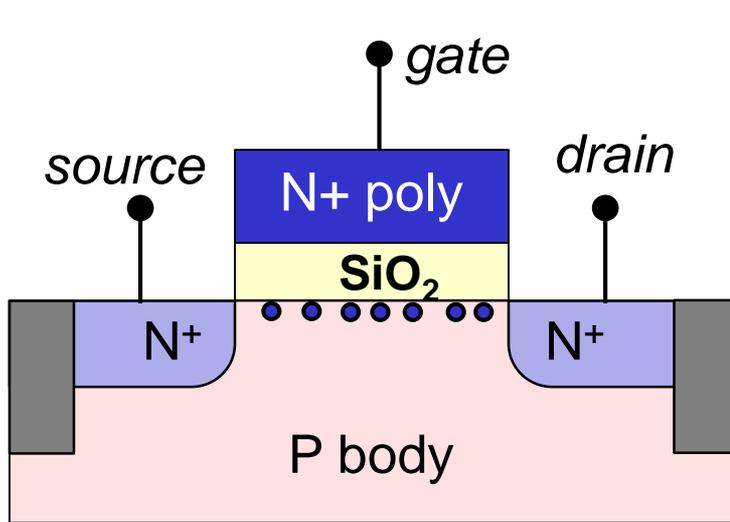
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Adapted from Modern Semiconductor Devices for Integrated Circuits, Chenming Hu, 2010



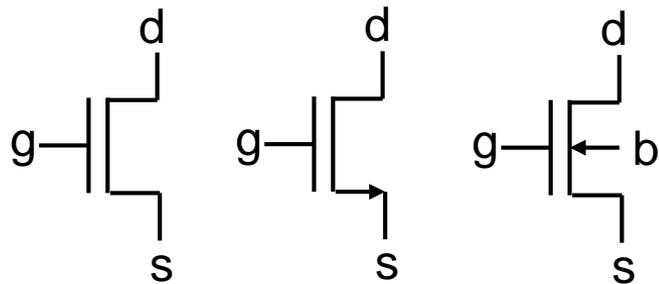
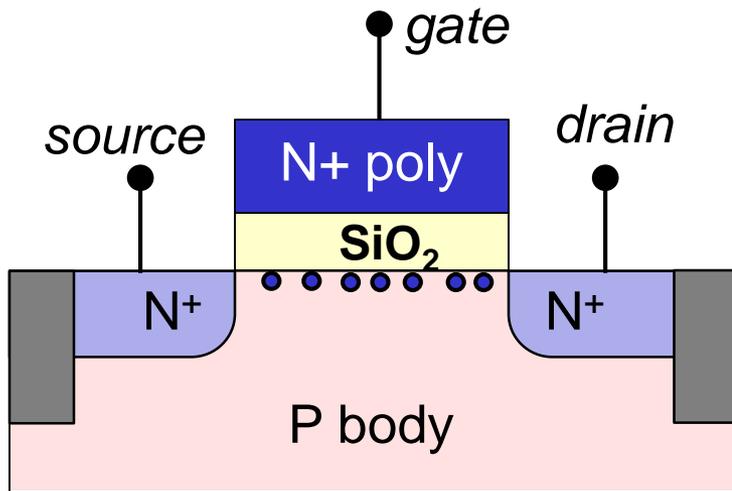
MOSFET Transistor



- Conduction between source and drain is controlled by gate voltage which induces and modulates inversion charge density
 - Inversion layer in a MOS transistor is often referred to as the [channel](#)
- MOS: Metal-Oxide-Semiconductor structure
- FET: Field Effect Transistor – conduction controlled by electric field

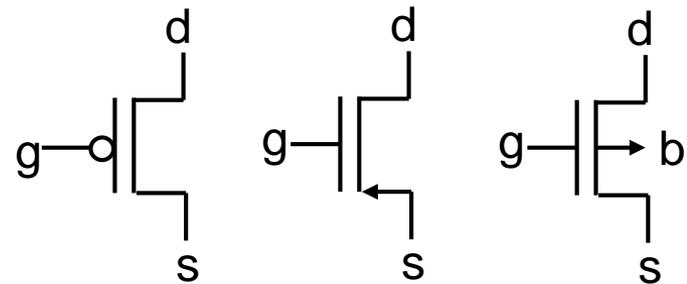
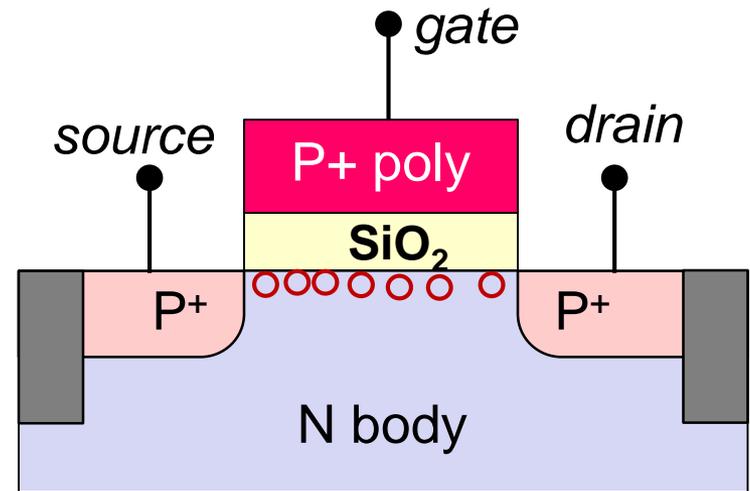
Complementary (CMOS) Technology

NFET or NMOS transistor



- P body normally connected to most negative voltage (0V, gnd)
- By definition: $V_s < V_d$

PFET or PMOS transistor



- N body normally connected to most positive voltage (V_{DD})
- By definition: $V_s > V_d$ 3

Surface Mobility

- Current through the channel of an MOS transistor is a function of electron (or hole) mobility in the inversion region
 - in a MOS transistor, we care about mobility in inversion layer
 - i.e., mobility near boundary between Si and SiO₂
- Surface mobility (μ_{ns}, μ_{ps}) is not same as bulk mobility
 - surface mobility is reduced by surface roughness scattering
- Surface mobility is a function of \vec{E}_{inv} average (vertical) electric field within the inversion charge layer

- For N⁺ poly gate NFET:
$$\vec{E}_{vert} \approx \frac{V_{gs} + V_t + 0.2V}{6 \cdot T_{oxe}}$$

- For P⁺ poly gate PFET:
$$\vec{E}_{vert} \approx -\frac{V_{gs} + 1.5V_t - 0.25V}{6 \cdot T_{oxe}}$$

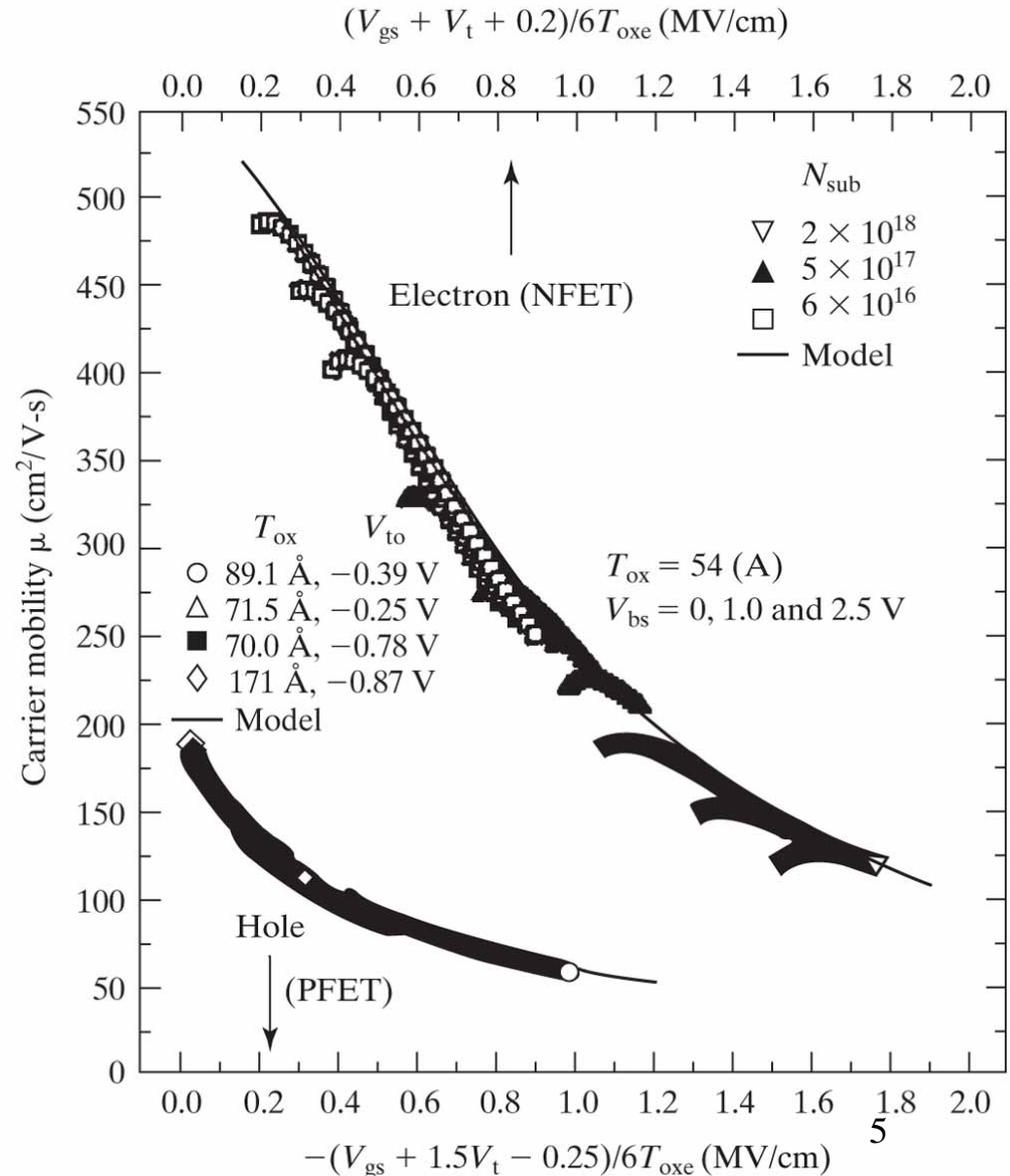
Empirical Mobility

- Empirically:

$$\mu_{ns} = \frac{540 \text{ cm}^2/\text{V}\cdot\text{s}}{1 + \left(\frac{V_{gs} + V_t + 0.2V}{5.4 \times 10^6 \cdot T_{oxe}}\right)^{1.85}}$$

$$\mu_{ps} = \frac{185 \text{ cm}^2/\text{V}\cdot\text{s}}{1 - \left(\frac{V_{gs} + 1.5V_t - 0.25V}{5.4 \times 10^6 \cdot T_{oxe}}\right)}$$

- Surface roughness scattering is greater at higher V_g , higher V_t and thinner T_{oxe}



Example: Surface Mobility

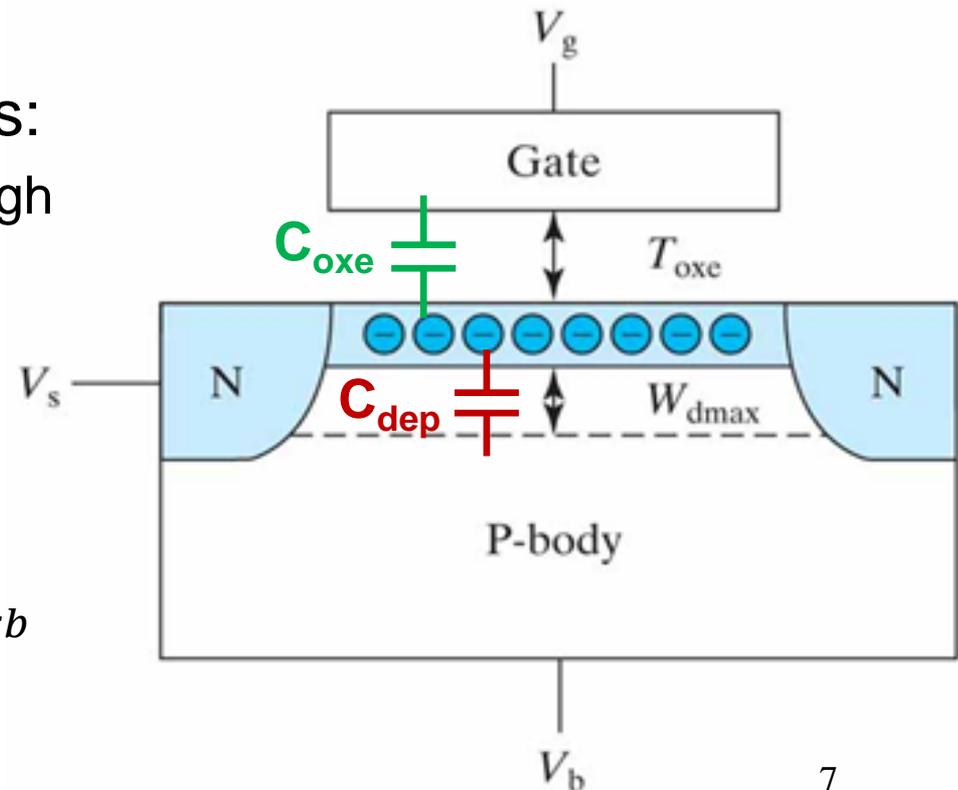
- What is the surface mobility at $V_{gs} = 1V$ in an N-channel MOSFET with $V_t = 0.3 V$ and $T_{oxe} = 2 nm$?

Body Effect

- Until now, we have assumed that the body and channel (inversion layer of MOS capacitor) are held at zero volts
- In MOS transistor, (channel potential) \geq (source potential)
- What happens if source (or channel) is positive with respect to body (i.e. $V_{sb} > 0$)?
- There are now two capacitors:
 - one from gate to channel through the oxide
 - a second from channel to body through depletion layer silicon
- Now:

$$Q_{inv} = -C_{oxe}(V_{gs} - V_t) + C_{dep} \cdot V_{sb}$$

$$\text{where } C_{dep} = \frac{\epsilon_s}{W_{dmax}}$$



Body Effect on Threshold Voltage

$$\begin{aligned}Q_{inv} &= -C_{oxe}(V_{gs} - V_t) + C_{dep} \cdot V_{sb} \\ &= -C_{oxe} \left(V_{gs} - \left(V_t + \frac{C_{dep}}{C_{oxe}} \cdot V_{sb} \right) \right) \\ &= -C_{oxe} \left(V_{gs} - V_t(V_{sb}) \right)\end{aligned}$$

$$V_t(V_{sb}) = V_{t0} + \frac{C_{dep}}{C_{oxe}} \cdot V_{sb} = V_{t0} + \alpha \cdot V_{sb}$$

$$\alpha = C_{dep}/C_{oxe} \approx (3T_{oxe})/W_{dmax}$$

- When source-body junction is reversed biased:
 - NFET V_t becomes more positive
 - PFET V_t becomes more negative
- This is known as **body effect** or **back gate bias effect**
 - α is the **body effect coefficient**

Uniform Body Doping

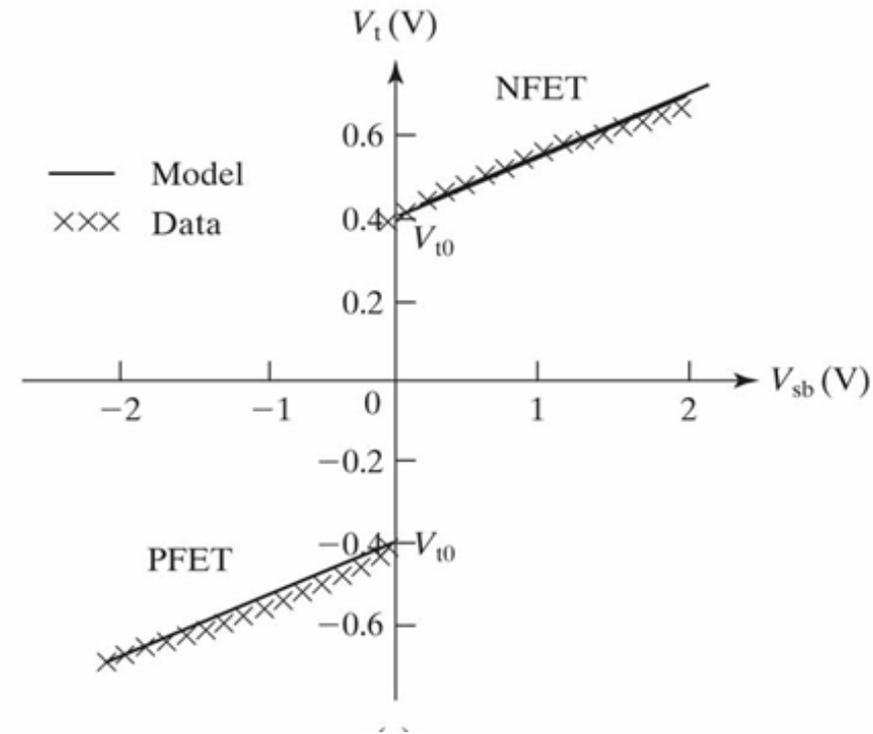
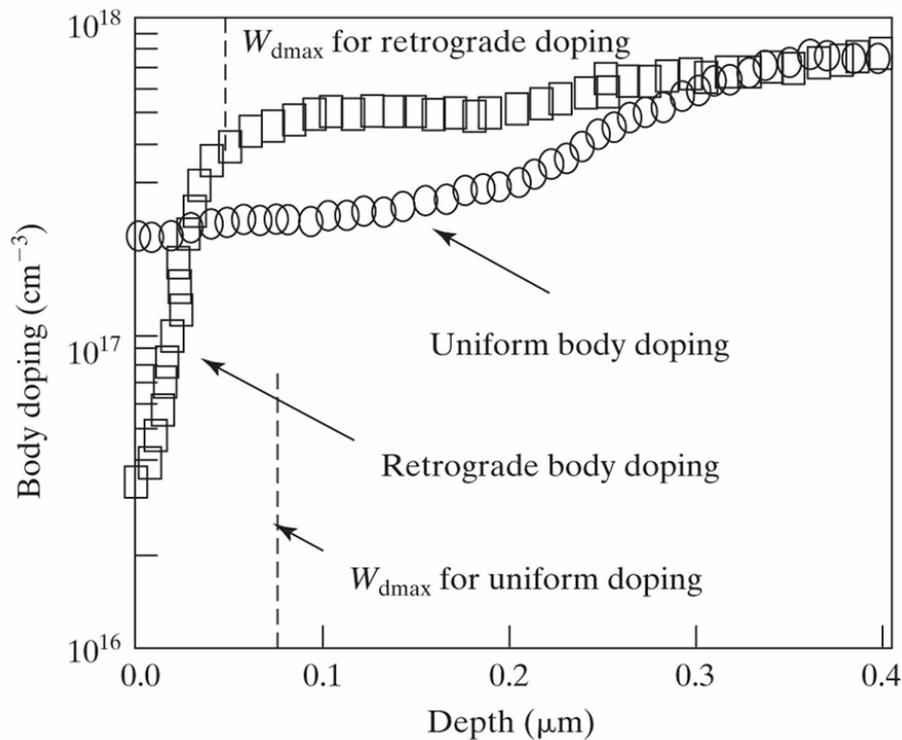
- Earlier generations MOS technology used uniform body doping
- Complicates expression for body effect because W_{dmax} is a function of V_{sb}
- V_t can be obtained by replacing $2\phi_B$ term (in expression for threshold voltage – previous lecture) with $(2\phi_B + V_{sb})$ which yields:

$$\begin{aligned} V_t &= V_{t0} + \frac{\sqrt{2q \cdot N_a \cdot \epsilon_s}}{C_{oxe}} (\sqrt{2\phi_B + V_{sb}} - \sqrt{2\phi_B}) \\ &= V_{t0} + \gamma (\sqrt{2\phi_B + V_{sb}} - \sqrt{2\phi_B}) \end{aligned}$$

- This is the “classical” formula for body-effect V_t variation
- γ is called **body-effect parameter**
 - sub-linear function of V_{sb}

Retrograde Body Doping

- Modern transistors employ steep retrograde body doping
 - light doping in surface layer with heavy doping underneath
- Reduces off-state leakage and improves surface mobility



- W_{dep} is now defined geometrically and independent of V_{sb}
- α is now constant \Rightarrow linear relation between V_t and V_{sb}

MOSFET IV – Very Small V_{ds}

- Consider MOS transistor with $V_{gs} > V_t, V_{ds} > 0$ but $V_{ds} \approx 0$
- Q_{inv} will be uniform along channel

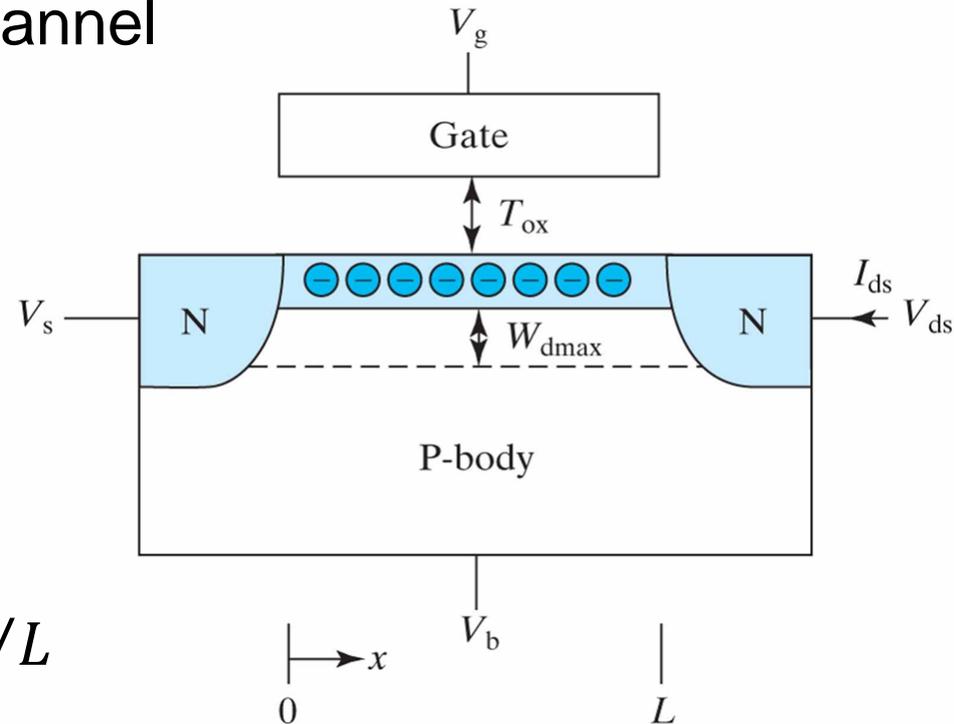
$$I_{ds} = W \cdot Q_{inv} \cdot v$$

where W is channel width

$$I_{ds} = W \cdot Q_{inv} \cdot \mu_{ns} \cdot \overrightarrow{E_x}$$

$$= W \cdot Q_{inv} \cdot \mu_{ns} \cdot V_{ds} / L$$

$$= W \cdot C_{oxe} (V_{gs} - V_t) \cdot \mu_{ns} \cdot V_{ds} / L$$



$$I_{ds} = \frac{W}{L} \cdot C_{oxe} \cdot \mu_{ns} \cdot (V_{gs} - V_t) \cdot V_{ds}$$

MOSFET IV – Beta

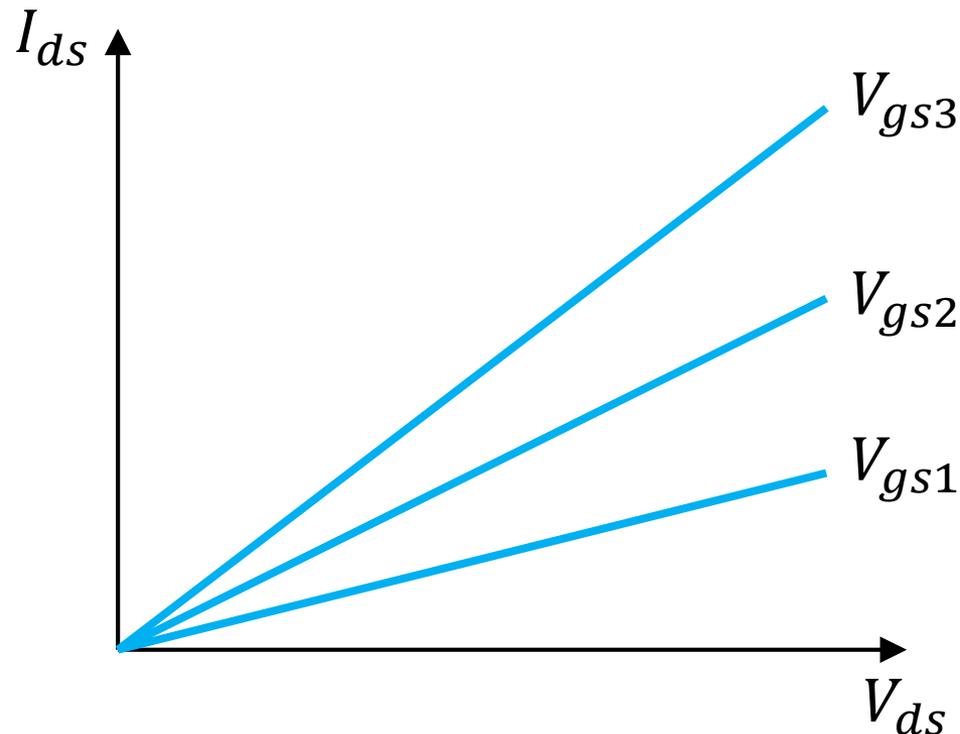
- MOS transistors are sometimes characterized by a gain factor β
- For **very small** V_{ds} :

$$I_{ds} = \frac{W}{L} \cdot C_{oxe} \cdot \mu_{ns} \cdot (V_{gs} - V_t) \cdot V_{ds} = \beta \cdot (V_{gs} - V_t) \cdot V_{ds}$$

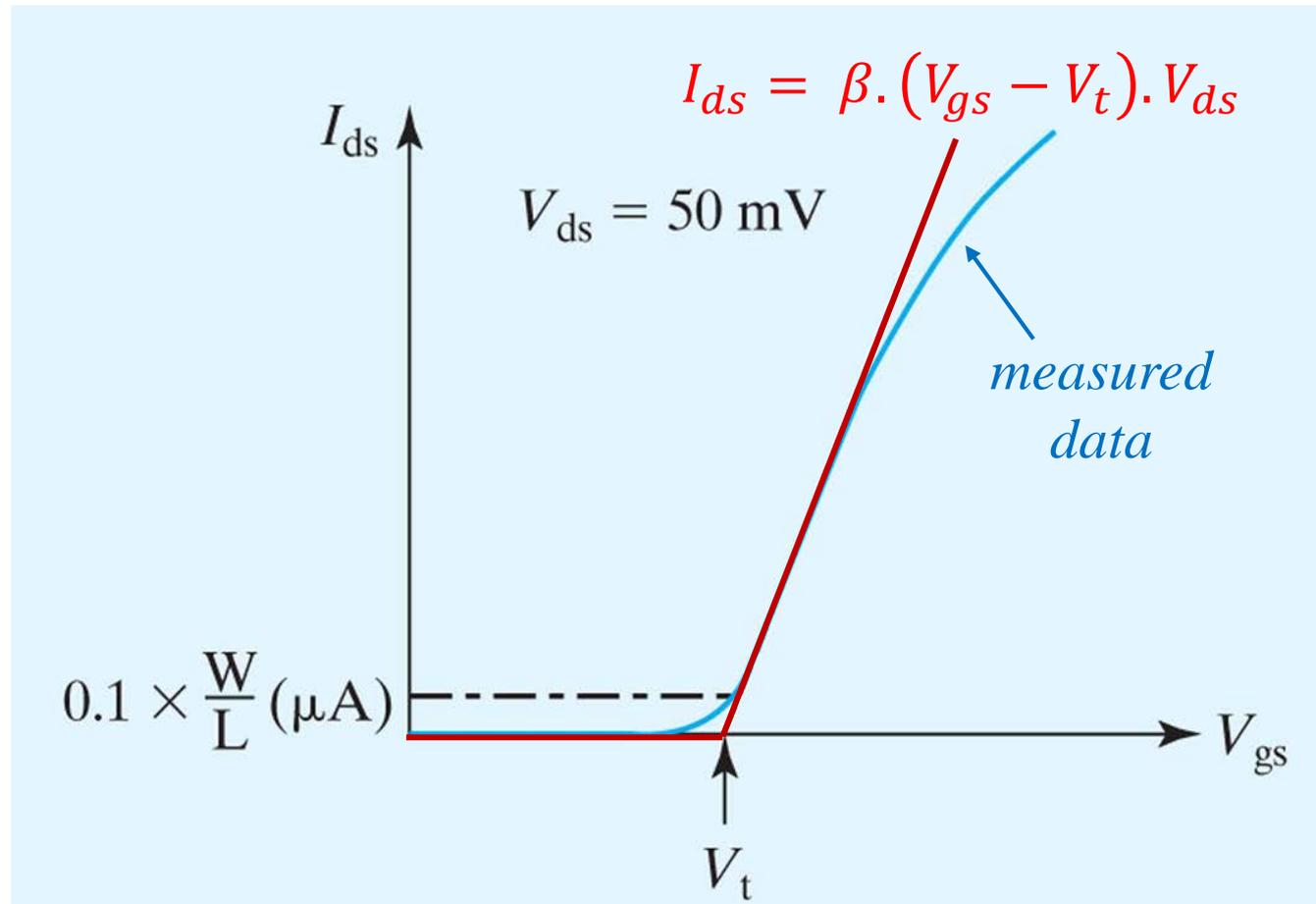
where $\beta = \frac{W}{L} \cdot C_{oxe} \cdot \mu_{ns}$ *β captures physical and material properties of the transistor*

MOSFET IV Characteristic – very small V_{ds}

- For **very small V_{ds}** : $I_{ds} = \beta \cdot (V_{gs} - V_t) \cdot V_{ds}$
 - If $V_{gs} < V_t$, $I_{ds} = 0$
 - If $V_{gs} > V_t$, MOSFET behaves as a linear resistor whose conductance is proportional to $(V_{gs} - V_t)$



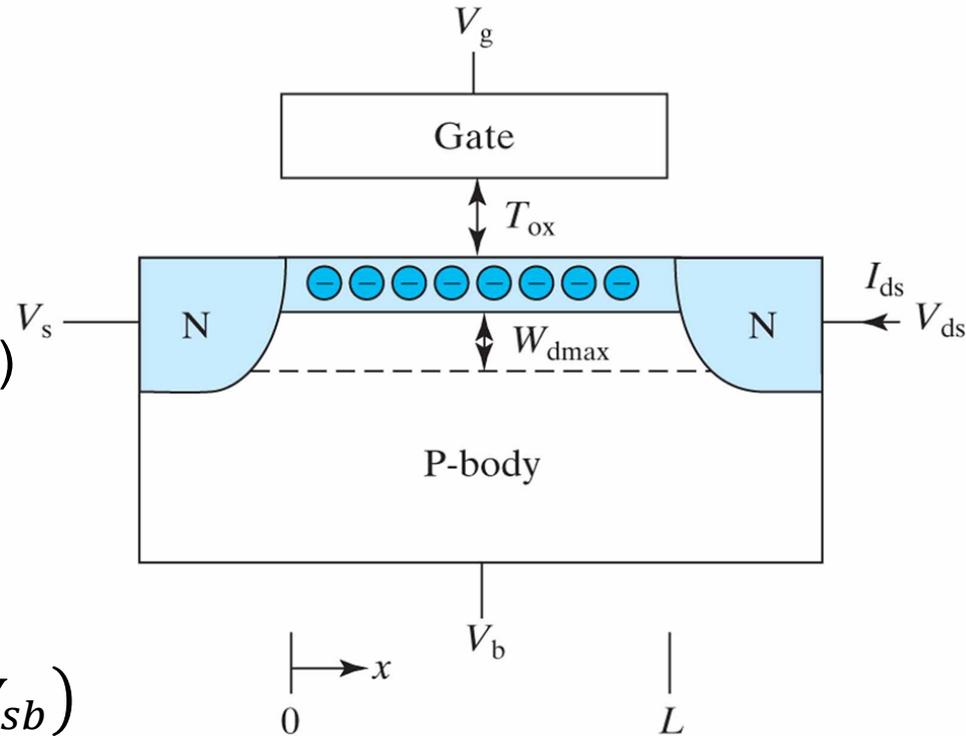
Measuring Threshold Voltage



- Empirical V_t can be determined by:
 - a) extrapolating I_{ds} vs. V_{gs} curve to $I_{ds} = 0$
 - b) measuring V_{gs} at which $I_{ds} = 0.1 \mu A \times (W/L)$

Inversion Charge in MOSFET

- Now consider MOSFET with $V_{ds} > 0$ (and no longer very small)
- Channel voltage V_c is now a function of x
 - $V_c(0) = V_s$ or $V_{cs}(0) = 0$
 - $V_c(L) = V_d$ or $V_{cs}(L) = V_{ds}$
- As we move toward the drain
 - less voltage across oxide (& more across depletion layer)
 - fewer electrons (less charge density) in inversion layer



$$Q_{inv} = -C_{oxe}(V_g - V_{t0} - \alpha \cdot V_{sb})$$

becomes:

$$Q_{inv}(x) = -C_{oxe}(V_{gc}(x) - V_{t0} - \alpha \cdot V_{cb}(x))$$

Bulk Charge Factor

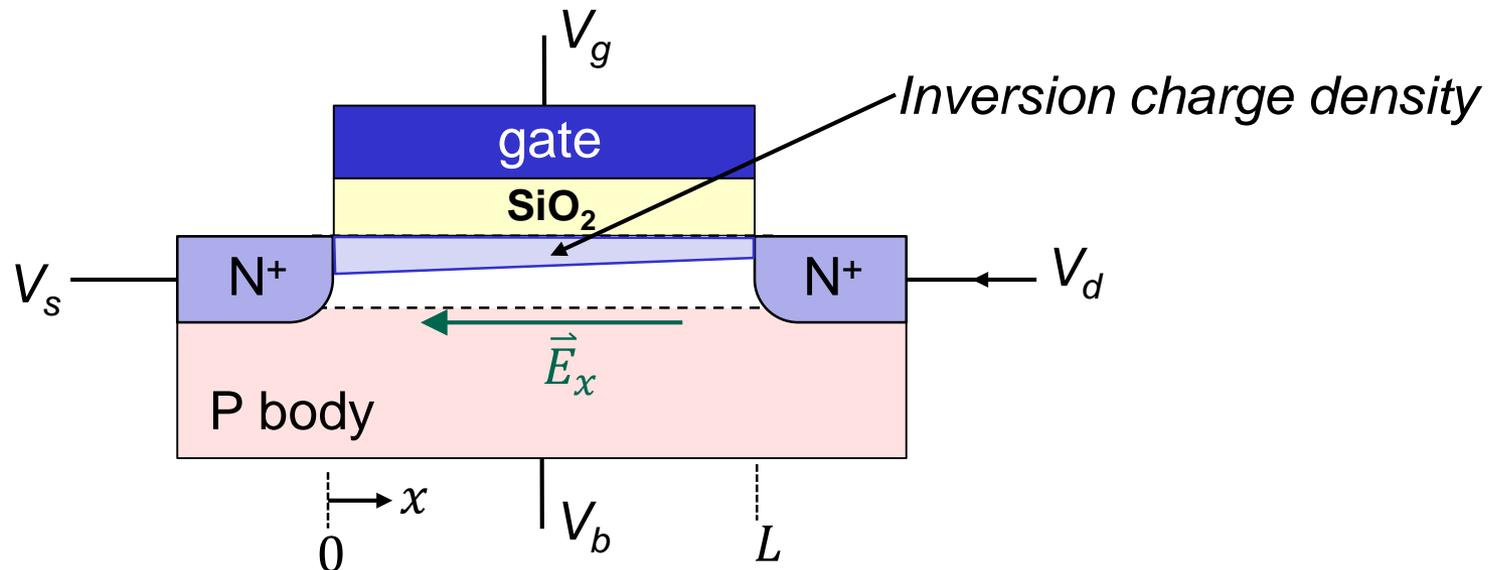
$$\begin{aligned}Q_{inv}(x) &= -C_{oxe}(V_{gc}(x) - V_{t0} - \alpha \cdot V_{cb}(x)) \\ &= -C_{oxe}(V_{gs} - V_{cs}(x) - V_{t0} - \alpha(V_{cs}(x) + V_{sb})) \\ &= -C_{oxe}(V_{gs} - V_{cs}(x) - (V_{t0} + \alpha V_{sb}) - \alpha \cdot V_{cs}(x)) \\ &= -C_{oxe}(V_{gs} - m \cdot V_{cs}(x) - V_t)\end{aligned}$$

where $m \equiv (1 + \alpha) = 1 + C_{dep}/C_{oxe} = 1 + 3(T_{oxe}/W_{dmax})$

- m is called **body effect factor** or **bulk charge factor**
- m is typically around 1.2
- First order models often use m=1

Channel Voltage Profile

- We know that $V_{cs}(0) = 0$ and $V_{cs}(L) = V_{ds}$
- What is $V_{cs}(x)$ for $0 < x < L$?
- Many simple analyses assume $V_{cs}(x) = x \cdot V_{ds} / L$
 - inversion charge density decreases linearly from source to drain
 - horizontal electric field $\vec{E}_x = dV_{cs}/dx$ is constant



- This cannot be correct as $I_{ds} \propto Q_{inv}(x)$. $\vec{E}_x(x)$ must be constant!
- $V_{cs}(x)$ is a much more complex function of x

MOSFET IV – Linear Region

- Fortunately, we can determine I_{ds} without knowing $V_{cs}(x)$
- We know $I_{ds} = W \cdot Q_{inv}(x) \cdot v(x) = W \cdot Q_{inv}(x) \cdot \mu_{ns} \cdot \vec{E}_x(x)$

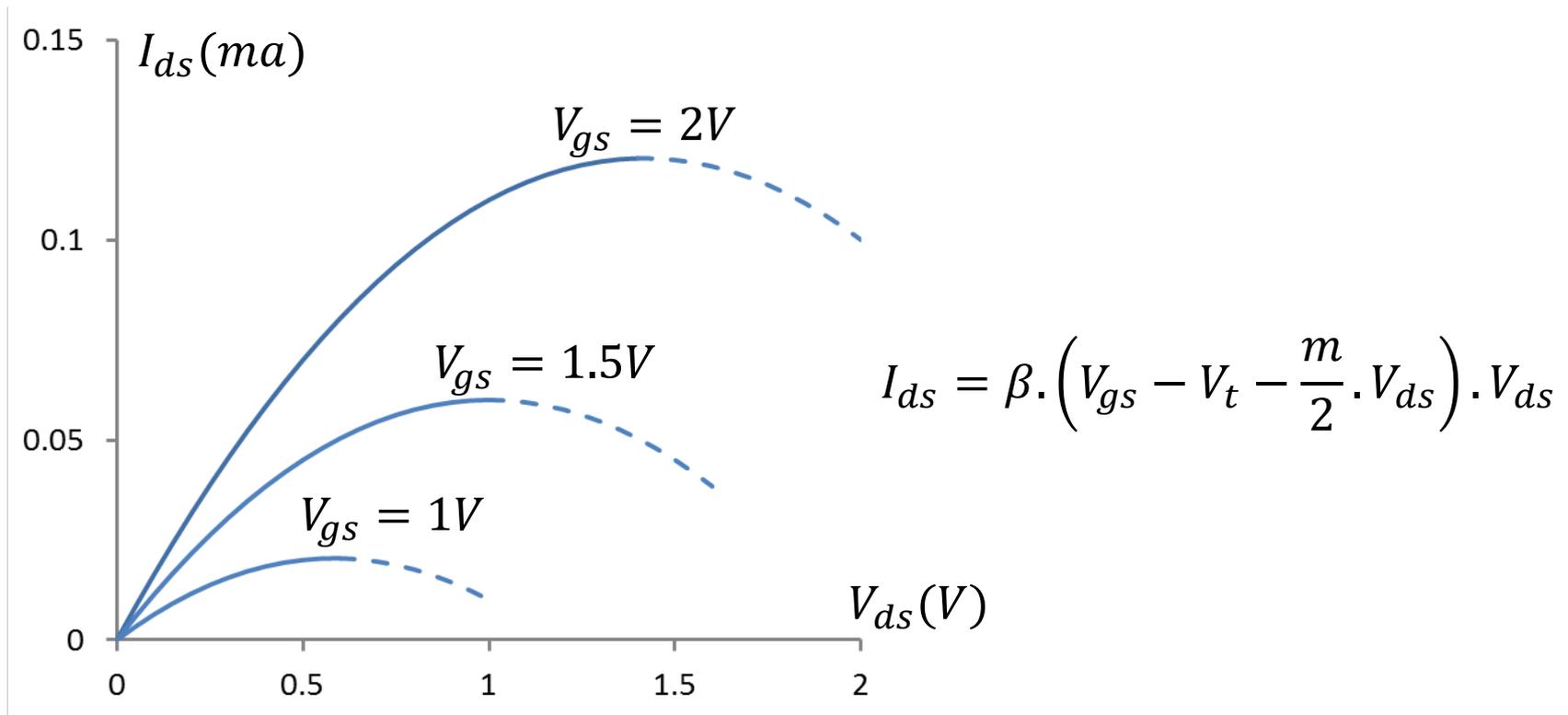
$$\text{so } I_{ds} \cdot dx = W \cdot C_{oxe} (V_{gs} - m \cdot V_{cs}(x) - V_t) \cdot \mu_{ns} \cdot \vec{E}_x(x) \cdot dx$$
$$= W \cdot C_{oxe} (V_{gs} - m \cdot V_{cs}(x) - V_t) \cdot \mu_{ns} \cdot dV_{cs}$$

$$\int_0^L I_{ds} \cdot dx = W \cdot C_{oxe} \cdot \mu_{ns} \int_0^{V_{ds}} (V_{gs} - m \cdot V_{cs}(x) - V_t) \cdot dV_{cs}$$

$$I_{ds} \cdot L = W \cdot C_{oxe} \cdot \mu_{ns} \left[(V_{gs} - V_t) \cdot V_{cs} - \frac{m \cdot V_{cs}^2}{2} \right]_{V_{cs}=0}^{V_{cs}=V_{ds}}$$

$$I_{ds} = \frac{W}{L} \cdot C_{oxe} \cdot \mu_{ns} \cdot \left(V_{gs} - V_t - \frac{m}{2} \cdot V_{ds} \right) \cdot V_{ds}$$
$$= \beta \cdot \left(V_{gs} - V_t - \frac{m}{2} \cdot V_{ds} \right) \cdot V_{ds}$$

MOSFET IV – Linear Region



- For small V_{ds} , I_{ds} increases linearly – behaves as a resistor
- As V_{ds} increases, charge in channel decreases
 - as a result: dI_{ds}/dV_{ds} decreases
- What happens when I_{ds} reaches its maximum ?

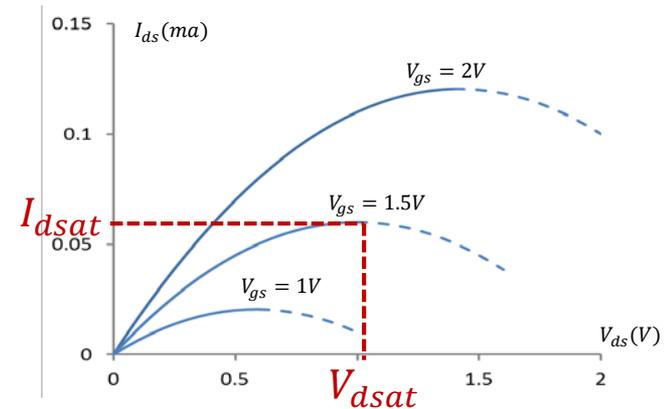
Drain Saturation Voltage

- At what point does I_{ds} reach its maximum?

$$I_{ds} = \beta \cdot \left(V_{gs} - V_t - \frac{m}{2} \cdot V_{ds} \right) \cdot V_{ds}$$

- Differentiating and setting to zero:

$$\frac{\partial I_{ds}}{\partial V_{ds}} = \beta \cdot (V_{gs} - V_t - m \cdot V_{ds}) = 0$$



- This gives:

$$V_{dsat} = \frac{V_{gs} - V_t}{m}$$

V_{dsat} is called the **drain saturation voltage**

- The drain current I_{ds} when $V_{ds} = V_{dsat}$ is given by:

$$I_{dsat} = I_{ds}(V_{dsat}) = \frac{W}{2m \cdot L} C_{oxe} \cdot \mu_{ns} \cdot (V_{gs} - V_t)^2$$

or
$$I_{dsat} = \frac{\beta}{2m} (V_{gs} - V_t)^2$$

I_{dsat} is called the **drain saturation current**

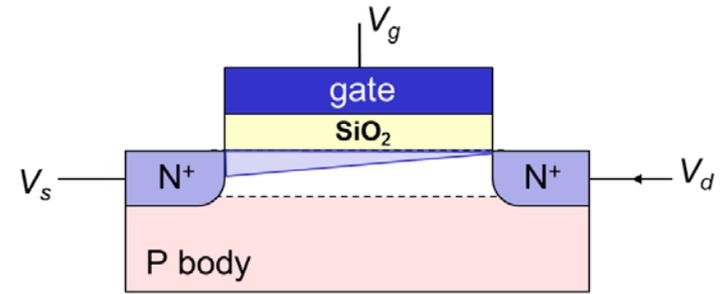
- Note that V_{dsat} and I_{dsat} are both functions of V_{gs}

Saturation

- What causes the current to “max-out” at V_{dsat} ?
- Charge in channel is: $Q_{inv}(x) = -C_{oxe}(V_{gs} - m \cdot V_{cs}(x) - V_t)$
- At saturation, $V_{cs}(x)$ at the drain end of channel is $V_{cs}(L) = V_{dsat}$

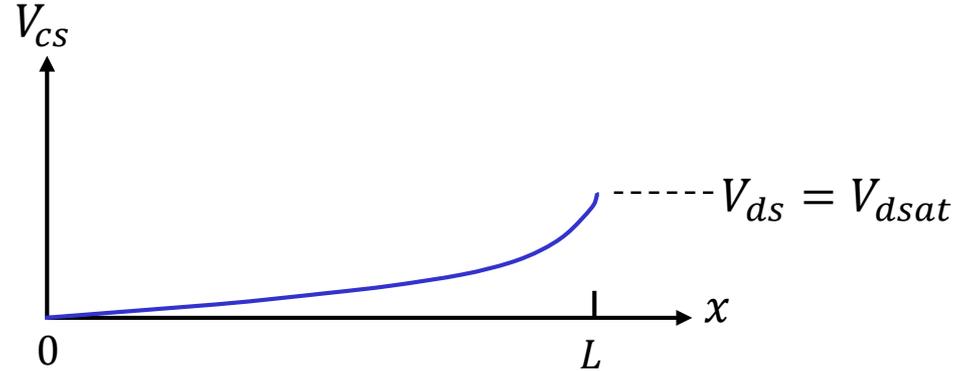
- Substituting into the equation above:

$$Q_{inv}(L) = 0 \text{ when } V_{ds} = V_{dsat}$$

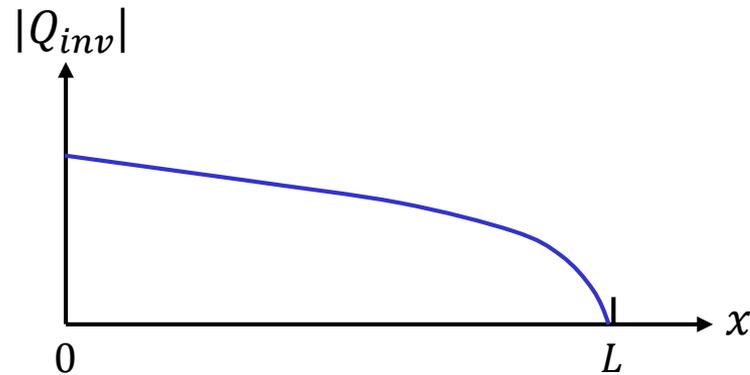


- i.e., the inversion charge density at the drain end of the channel has been reduced to zero!
- Disappearance of the inversion layer is called channel **pinch-off**

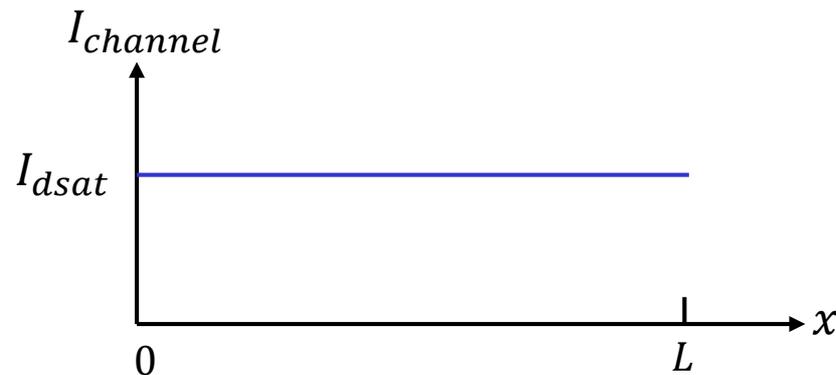
Voltage & Charge in Channel at Saturation



$$|Q_{inv}| = C_{ox}(V_g - m \cdot V_{cs} - V_t)$$

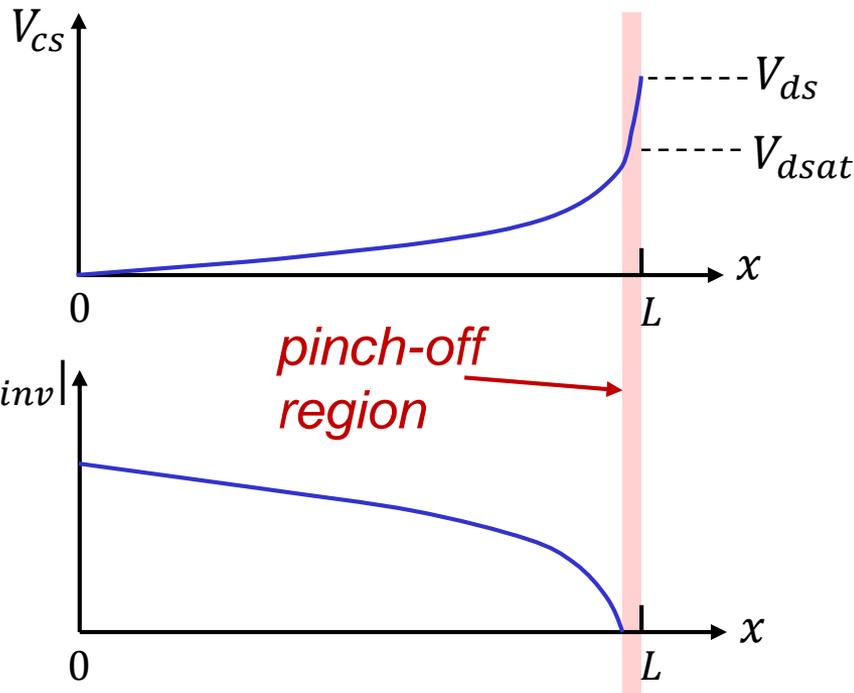


$$I_{channel} = \mu_{ns} \cdot Q_{inv} \cdot \frac{dV_{cs}}{dx}$$



Saturation Region

- What happens when V_{ds} is increased beyond V_{dsat} ?
- Pinch-off region (i.e. the region where the channel charge density is zero) now extends a short finite distance from the drain toward the source
- Pinch-off region is a high-field region across which the voltage $V_{ds} - V_{dsat}$ is dropped



Note that because the pinch-off region is very small, $V_{cs}(x)$ and $Q_{inv}(x)$ are almost identical to what they were when $V_{ds} = V_{dsat}$

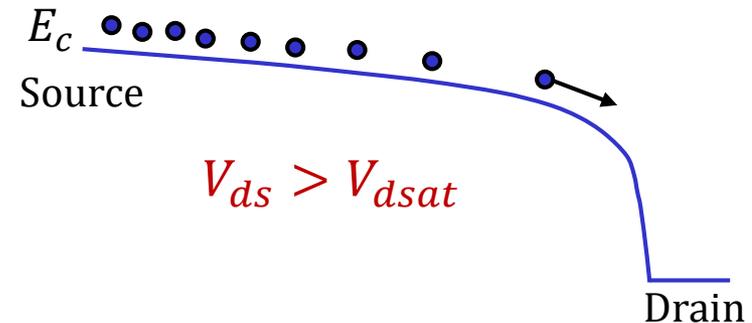
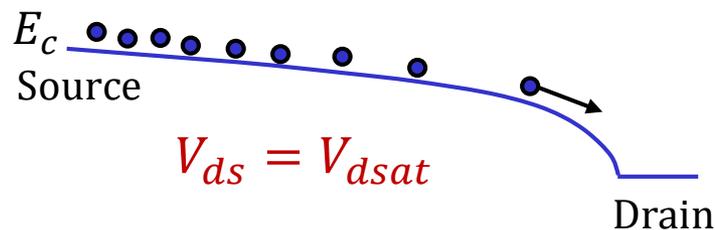
$$\text{But } I_{channel} = \mu_{ns} \cdot Q_{inv} \cdot \frac{dV_{cs}}{dx}$$

As a result:

$$I_{ds} = I_{dsat} \quad \text{for } V_{ds} \geq V_{dsat}$$

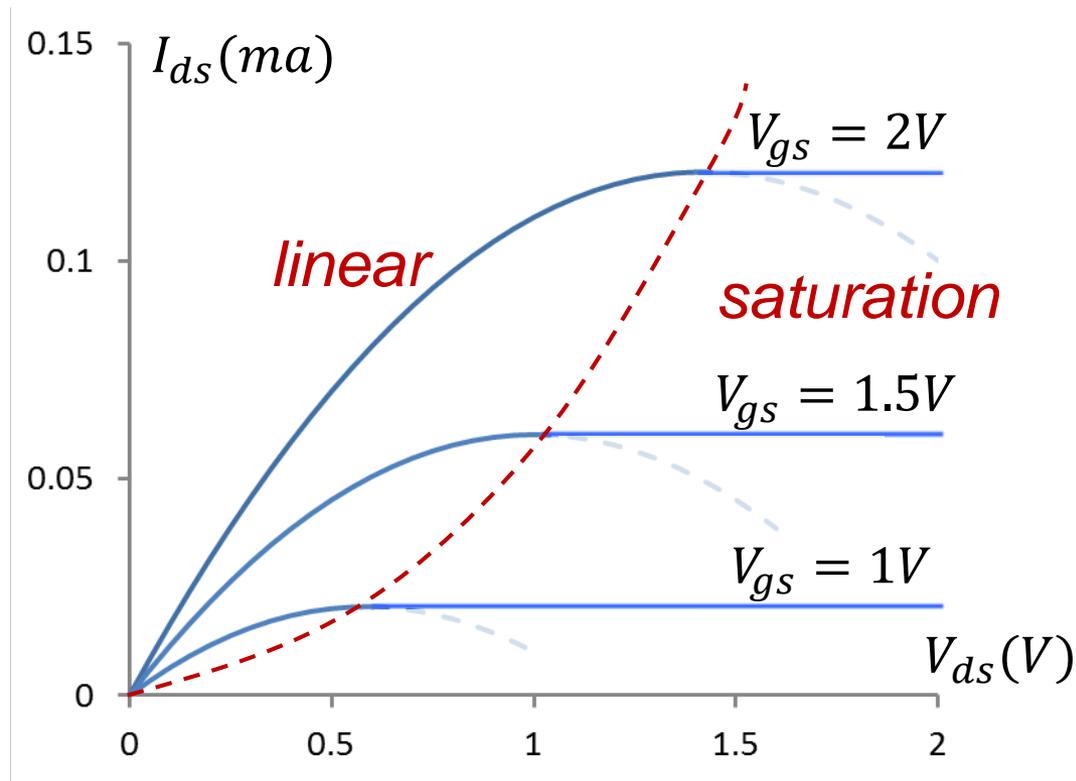
Saturation Region

- MOSFET acts as a constant current source in saturation
- How can current flow through pinch-off region if there is no inversion charge?
- Pinch-off region is a depletion region
- Depletion region is not a barrier to current flow
 - Current can flow across depletion region if there is an electric field and there is a source of carriers (as in PN diode)
 - When electrons reach pinch-off region, they are swept across this region by the strong electric field



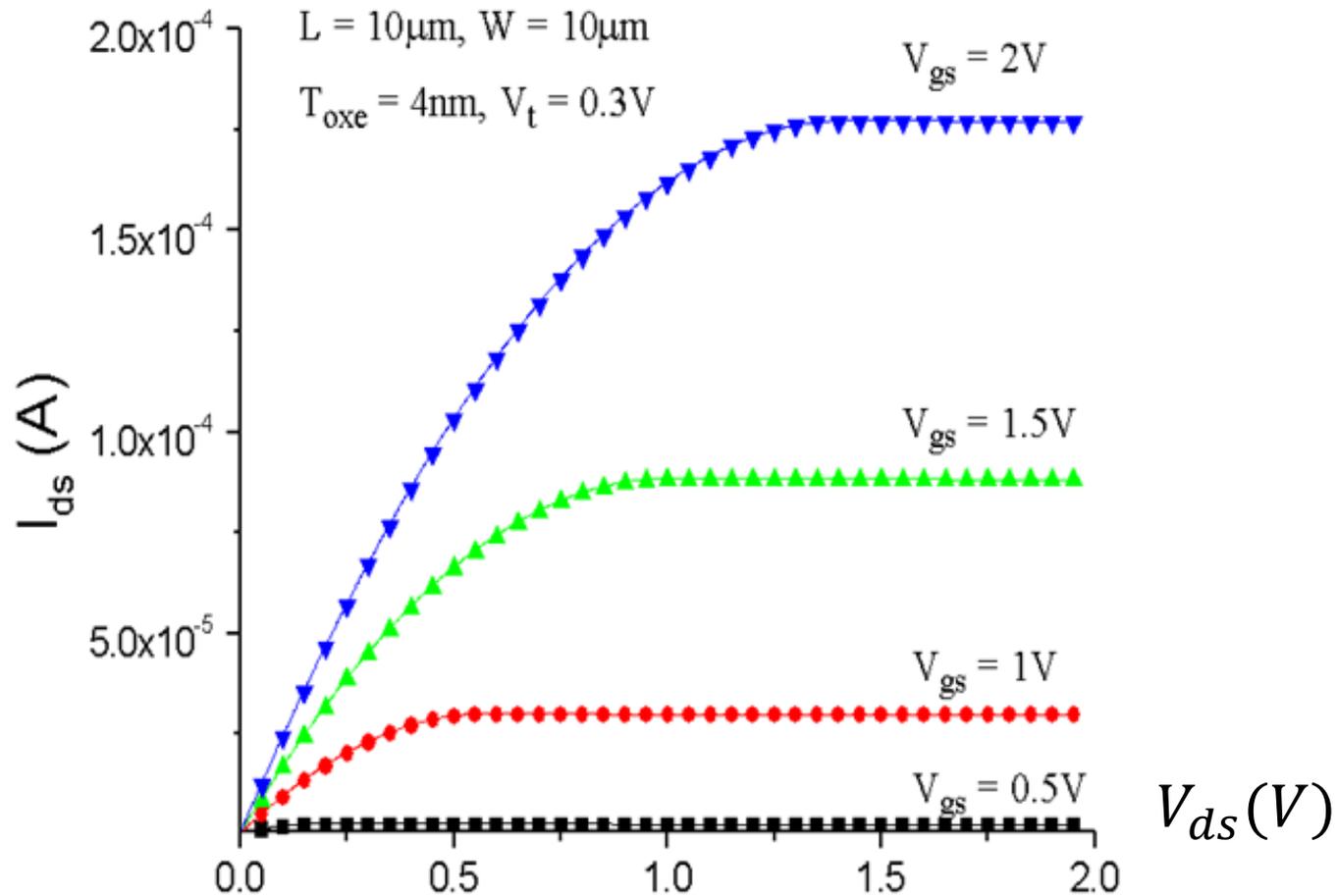
- Waterfall analogy
 - flow down waterfall is independent of height of waterfall

MOSFET IV – Linear & Saturation Regions



- For small V_{ds} , I_{ds} increases linearly – behaves as a resistor
- As V_{ds} increases, charge in channel decreases
 - as a result: dI_{ds}/dV_{ds} decreases
- When V_{ds} reaches $V_{dsat} = (V_{gs} - V_t)/m$, pinch-off occurs
 - $I_{ds} = I_{dsat}$ - transistor behaves as constant current source

MOSFET IV – Long Channel Model



- Equations we have developed sometimes known as first-order model or Shockley model (esp. with $m=1$), or long-channel model
- Accurately describe behavior of long-channel MOSFETs

Transconductance

- In saturation region, MOSFET behaves as a voltage controlled constant current source
- Convenient to describe “gain” of the transistor in terms of its transconductance:

$$g_m = \frac{\partial I_{ds}}{\partial V_{gs}} = \frac{W}{m \cdot L} \cdot C_{oxe} \cdot \mu_{ns} (V_{gs} - V_t)$$

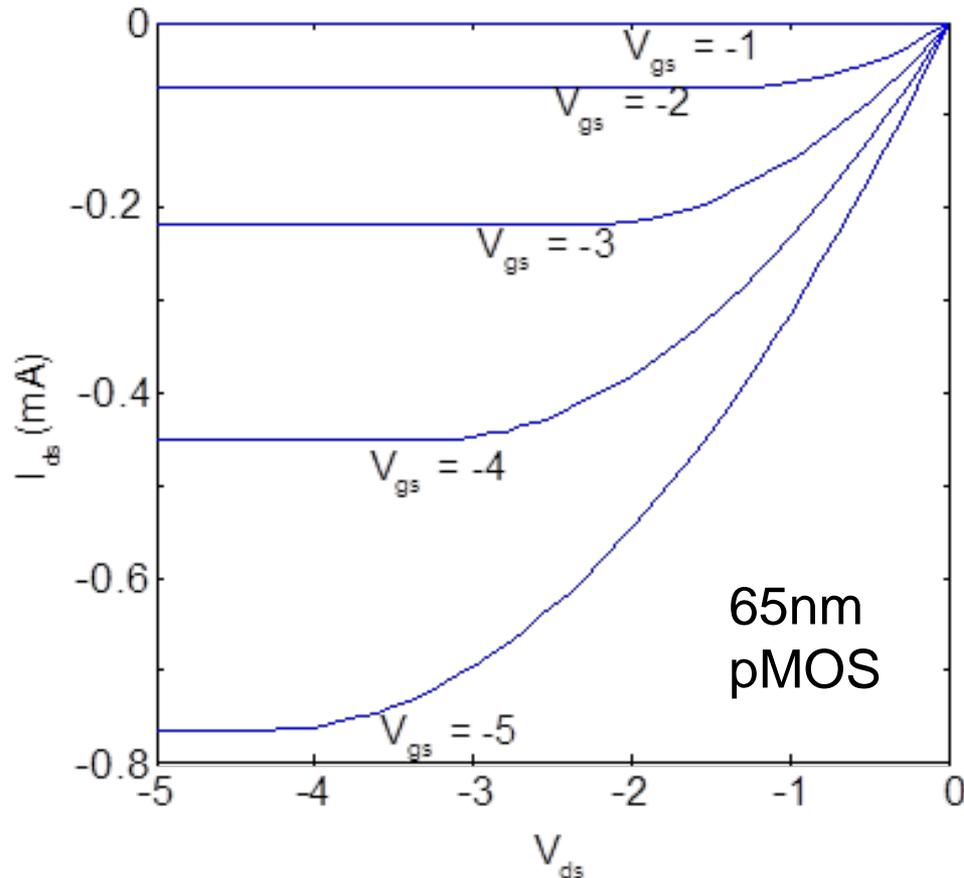
$$g_m = \frac{\beta}{m} (V_{gs} - V_t)$$

Summary of **NMOS** Long Channel Model

Cut-Off	$V_{gs} < V_t$	$I_{ds} = 0$
Linear	$V_{gs} > V_t$ $V_{ds} < V_{dsat}$	$I_{ds} = \beta \cdot \left(V_{gs} - V_t - \frac{m}{2} \cdot V_{ds} \right) \cdot V_{ds}$
Saturation	$V_{gs} > V_t$ $V_{ds} > V_{dsat}$	$I_{ds} = \frac{\beta}{2m} (V_{gs} - V_t)^2$ $g_m = \frac{\beta}{m} (V_{gs} - V_t)$

PMOS Long Channel Model

- All dopings and voltages are inverted for PMOS
 - Source is the more positive terminal
 - V_{gs} , V_{ds} , V_{dsat} , V_t and I_{ds} are all ≤ 0



- Mobility determined by holes
 - μ_{ps} typically 2-3x lower than that of electrons μ_{ns}

$$\beta = \frac{W}{L} \cdot C_{oxe} \cdot \mu_{ps}$$

- PMOS transistors must be 2-3 times wider to provide the same drive current

Summary of PMOS Long Channel Model

- Note that in PMOS transistor:

V_{gs} , V_{ds} , V_{dsat} , V_t and I_{ds} are all ≤ 0

Cut-Off	$V_{gs} > V_t$	$I_{ds} = 0$
Linear	$V_{gs} < V_t$ $V_{ds} > V_{dsat}$	$I_{ds} = -\beta \cdot \left(V_{gs} - V_t - \frac{m}{2} \cdot V_{ds} \right) \cdot V_{ds}$
Saturation	$V_{gs} < V_t$ $V_{ds} < V_{dsat}$	$I_{ds} = \frac{-\beta}{2m} (V_{gs} - V_t)^2$ $g_m = \frac{-\beta}{m} (V_{gs} - V_t)$

Example:

- An ideal NMOS transistor has the following parameters:
 - $W = 50\mu m, L = 5\mu m$
 - $T_{oxe} = 25\text{ nm}$
 - $W_{dmax} = 375\text{ nm}$
 - $V_t = 0.4\text{ V}$
 - $\mu_{ns} = 350\text{ cm}^2/\text{V}\cdot\text{s}$
- Determine:
 - a) V_{dsat}, I_{dsat} and g_{msat} at $V_{gs} = 1, 2$ and 3 V
 - b) Sketch I_{ds} vs. V_{ds} curves for $0 < V_{ds} < 3\text{ V}$ and $V_{gs} = 1, 2$ and 3 V

Non-Ideal I-V Effects

- We will now examine non-ideal effects that detract from the behavior described by long-channel model:

- Velocity Saturation

- Channel Length Modulation

- Body Effect (we already covered this)

- variation of threshold voltage with source-body bias

$$V_t = V_{t0} + \alpha \cdot V_{sb} \quad \text{where} \quad \alpha = (m - 1) \approx 3 \cdot C_{oxe} / W_{dmax}$$

- Sub-threshold Current

- Drain Induced Barrier Lowering (DIBL)

Velocity Saturation

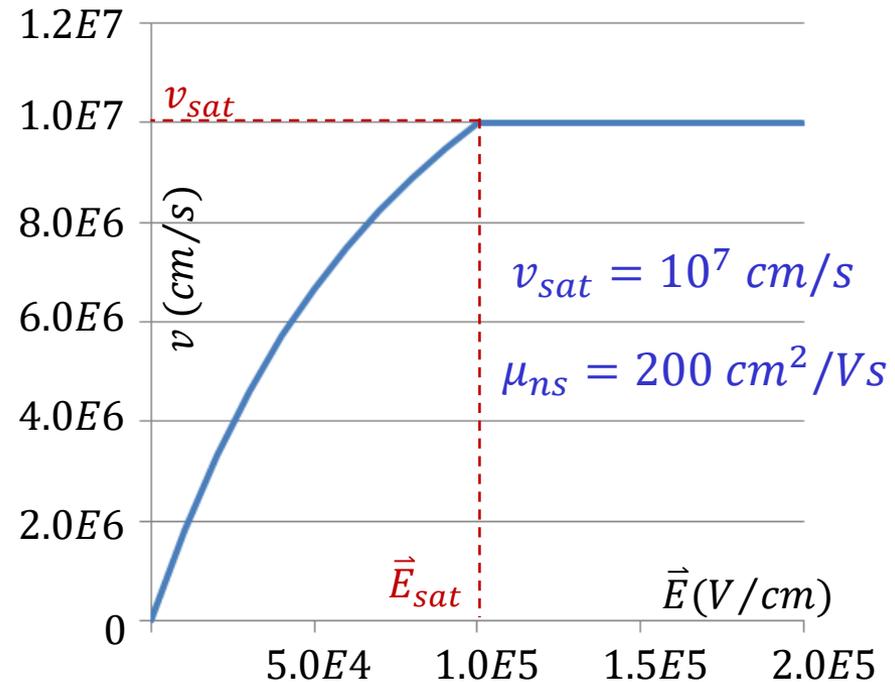
- In short channel length devices, electric field can be very high – leads to velocity saturation
 - particularly when device is in saturation and very high field exists in pinch-off region

- Empirically model velocity saturation as:

$$v = \frac{\mu_{ns} \cdot \vec{E}}{1 + \vec{E} / \vec{E}_{sat}} \quad \text{for } \vec{E} \leq \vec{E}_{sat}$$

$$v = v_{sat} \quad \text{for } \vec{E} \geq \vec{E}_{sat}$$

- where $\vec{E}_{sat} \equiv 2 \cdot v_{sat} / \mu_{ns}$



MOS IV Model with Velocity Saturation

- Re-evaluating expressions for I_{ds} (linear) with saturation limited mobility gives:

$$I_{ds} = \frac{\beta \left(V_{gs} - V_t - \frac{m}{2} \cdot V_{ds} \right) \cdot V_{ds}}{1 + V_{ds} / (\vec{E}_{sat} \cdot L)} \quad \text{for } V_{ds} < V_{dsat}, v < v_{sat}$$

$$= \frac{\text{long-channel } I_{ds}}{1 + V_{ds} / (\vec{E}_{sat} \cdot L)} \quad \text{usually valid in linear range}$$

- At onset of pinch-off and velocity saturation:

$$\begin{aligned} I_{ds} &= I_{ds}(L) = W \cdot Q_{inv}(L) \cdot v(L) \\ &= W \cdot C_{oxe} \cdot (V_{gs} - V_t - m \cdot V_{dsat}) \cdot v_{sat} \end{aligned}$$

Saturation Voltage with Velocity Saturation

- Equating these two expressions for I_{ds} at $V_{ds} = V_{dsat}$ gives:

$$\frac{1}{V_{dsat}} = \frac{m}{V_{gs} - V_t} + \frac{1}{\vec{E}_{sat} \cdot L}$$

- V_{dsat} is reduced by velocity saturation
- V_{dsat} is less than smaller of:
 - long-channel V_{dsat} and
 - that V_{ds} that would give an average electric field of \vec{E}_{sat} along the length of the channel
 - (same math as two resistors in parallel)

Saturation Current with Velocity Saturation

- Substituting eqn. for V_{dsat} into (linear) expression for I_{ds} :

$$I_{dsat} = \frac{W}{2m \cdot L} \cdot C_{oxe} \cdot \mu_{ns} \cdot \frac{(V_{gs} - V_t)^2}{1 + \frac{V_{gs} - V_t}{m \cdot \vec{E}_{sat} \cdot L}}$$
$$= \frac{\text{long-channel } I_{dsat}}{1 + \frac{V_{gs} - V_t}{m \cdot \vec{E}_{sat} \cdot L}} = \frac{\beta}{2} (V_{gs} - V_t) \cdot V_{dsat}$$

- Note that for long channel case or low V_{gs} ,

$$\vec{E}_{sat} \cdot L \gg (V_{gs} - V_t)$$

- this reverts simply to long channel model

Short Channel MOSFETs

- For the very short channel case: $\vec{E}_{sat} \cdot L \ll (V_{gs} - V_t)$

$$V_{dsat} \approx \vec{E}_{sat} \cdot L$$

$$I_{dsat} \approx W \cdot v_{sat} \cdot C_{oxe} (V_{gs} - V_t - m \cdot \vec{E}_{sat} \cdot L)$$

- Note that I_{dsat} is no longer proportional to $1/L$
- And $I_{dsat} \propto (V_{gs} - V_t)$ rather than $(V_{gs} - V_t)^2$
- In short-channel devices, pinch-off region becomes a velocity saturation region
- Velocity saturation is a serious limitation on the I_{on} of short channel MOSFETs

Example: Velocity Saturation

At $V_{gs} = 1.8$ V, what is the V_{dsat} and I_{dsat} of an NFET with $W = 1$ μm , $T_{oxe} = 3$ nm, $\mu_{ns} = 200$ cm^2/Vs , $v_{sat} = 8 \times 10^6$ cm/s , $V_t = 0.25$ V, and $W_{dmax} = 45$ nm for:

(a) $L = 10$ μm

(b) $L = 1$ μm

(c) $L = 0.1$ μm

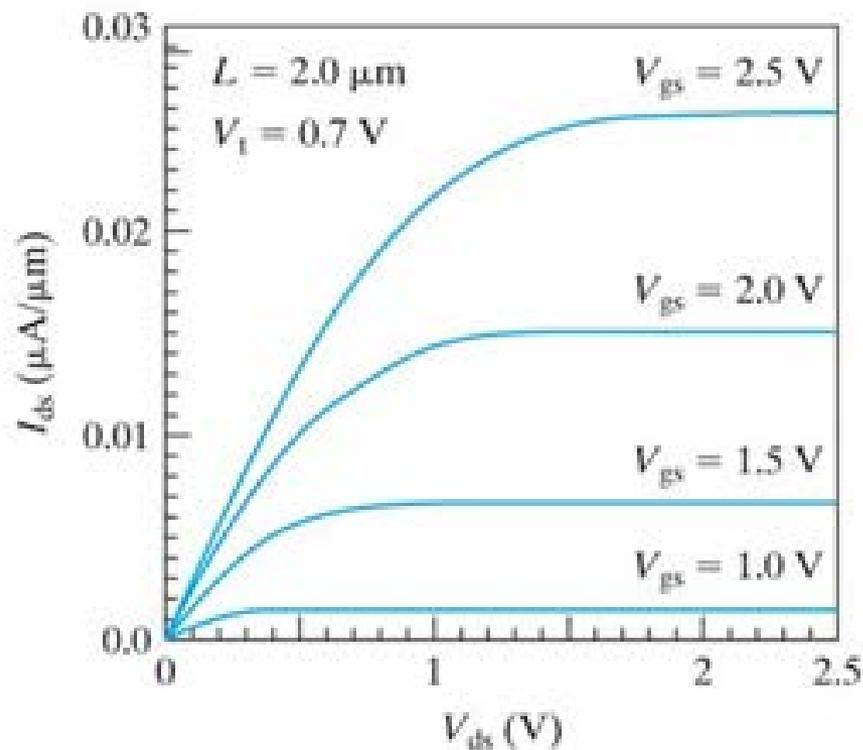
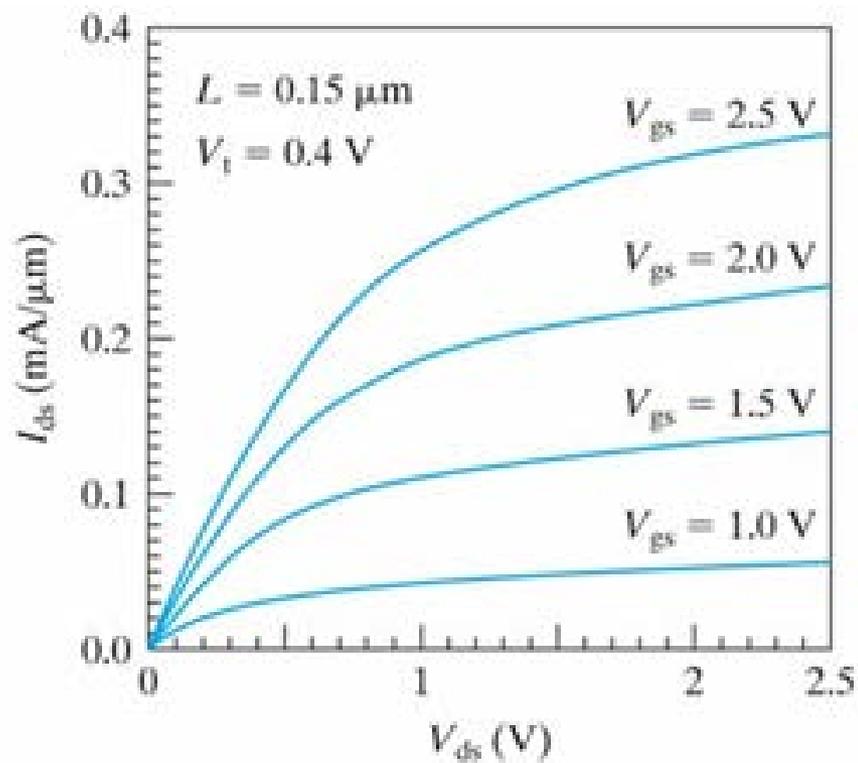
(d) $L = 0.05$ μm ?

Compare to Long-Channel Model

Using data from previous example:

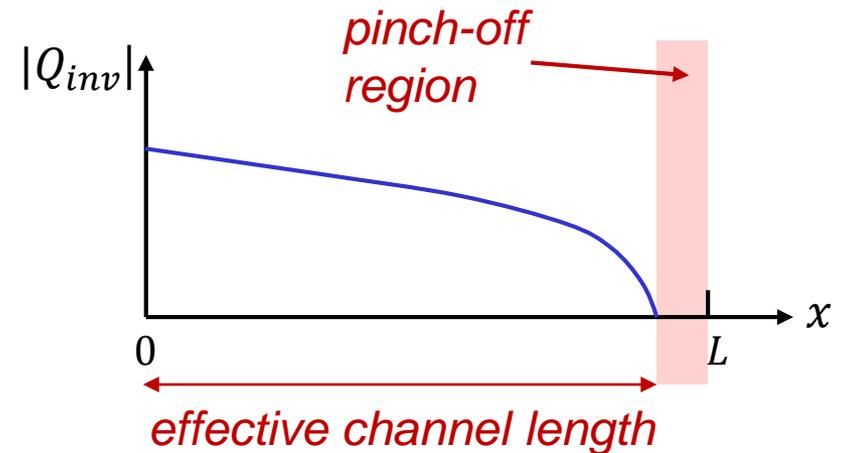
L (μm)	Long-channel		with Velocity Saturation	
	V_{dsat} (V)	I_{dsat} (μA)	V_{dsat} (V)	I_{dsat} (μA)
10	1.29	23.1	1.27	22.8
1	1.29	231	1.11	200
0.1	1.29	2310	0.494	882
0.05	1.29	4620	0.305	1090

Compare Long & Short Channel MOSFETs



Channel Length Modulation

- Until now, we have assumed that length of pinch-off region does not significantly reduce length of channel
- In short channel length devices, as V_{ds} increases beyond V_{sat} , effective channel length decreases leading to an increase in I_{ds}



- I_{ds} increases with increasing V_{ds} - even in saturation
- Can be modeled as:

$$I_{ds} = \frac{\beta}{2m} (V_{gs} - V_t)^2 \cdot (1 + \lambda \cdot V_{ds}) = I_{dsat} (1 + \lambda \cdot V_{ds})$$

- where λ is channel length modulation parameter
 - units are V^{-1}

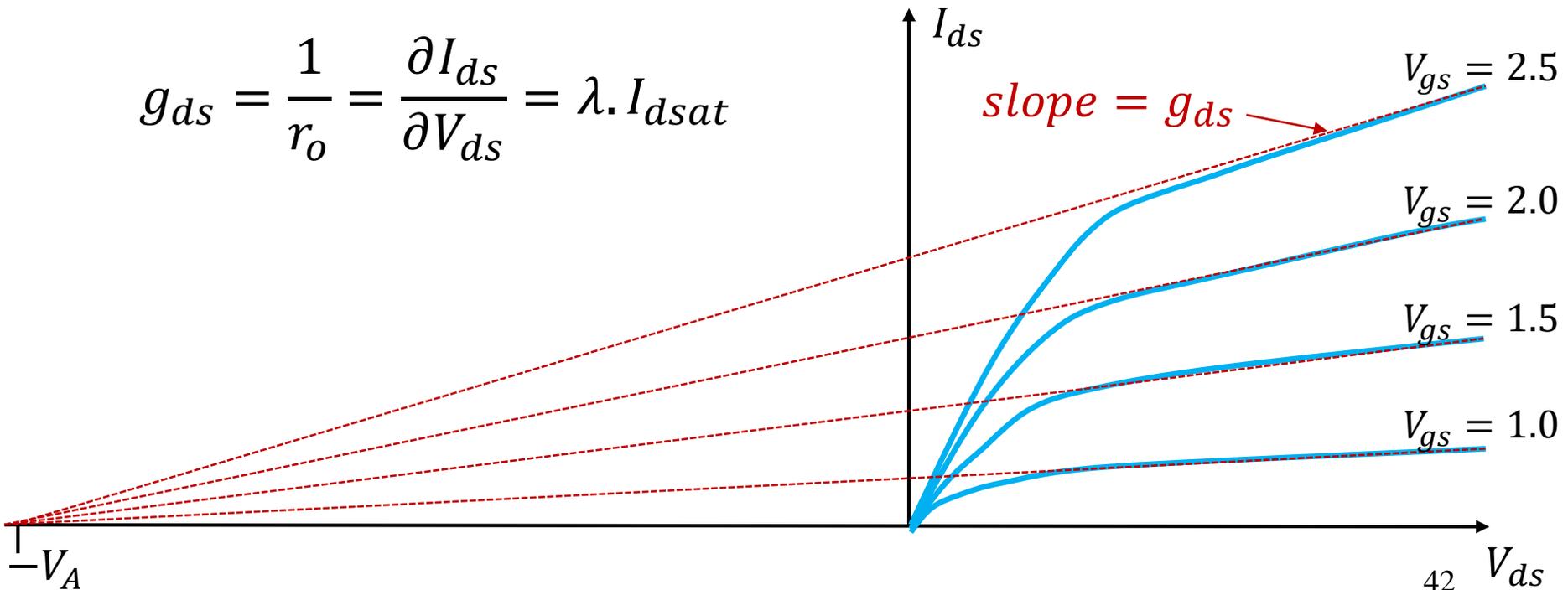
Output Conductance

- Like bipolar transistor, channel length modulation can also be modeled as an Early effect:

$$I_{ds} = V_{dsat} (1 + V_{ds}/V_A) \quad \text{where Early voltage } V_A = 1/\lambda$$

- In saturation, small signal output conductance:

$$g_{ds} = \frac{1}{r_o} = \frac{\partial I_{ds}}{\partial V_{ds}} = \lambda \cdot I_{dsat}$$



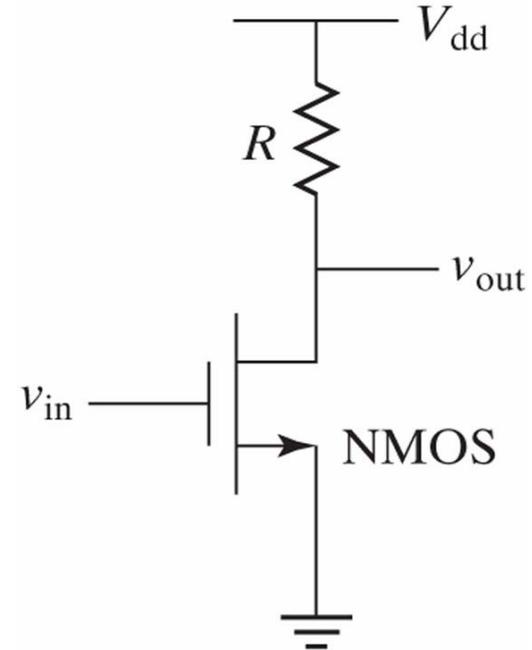
Example: MOS Amplifier

- An MOS transistor operates in the saturation region. A **small signal** input, v_{in} is applied.

$$\begin{aligned}i_{ds} &= g_m \cdot v_{gs} + g_{ds} \cdot v_{ds} \\ &= g_m \cdot v_{in} + g_{ds} \cdot v_{out}\end{aligned}$$

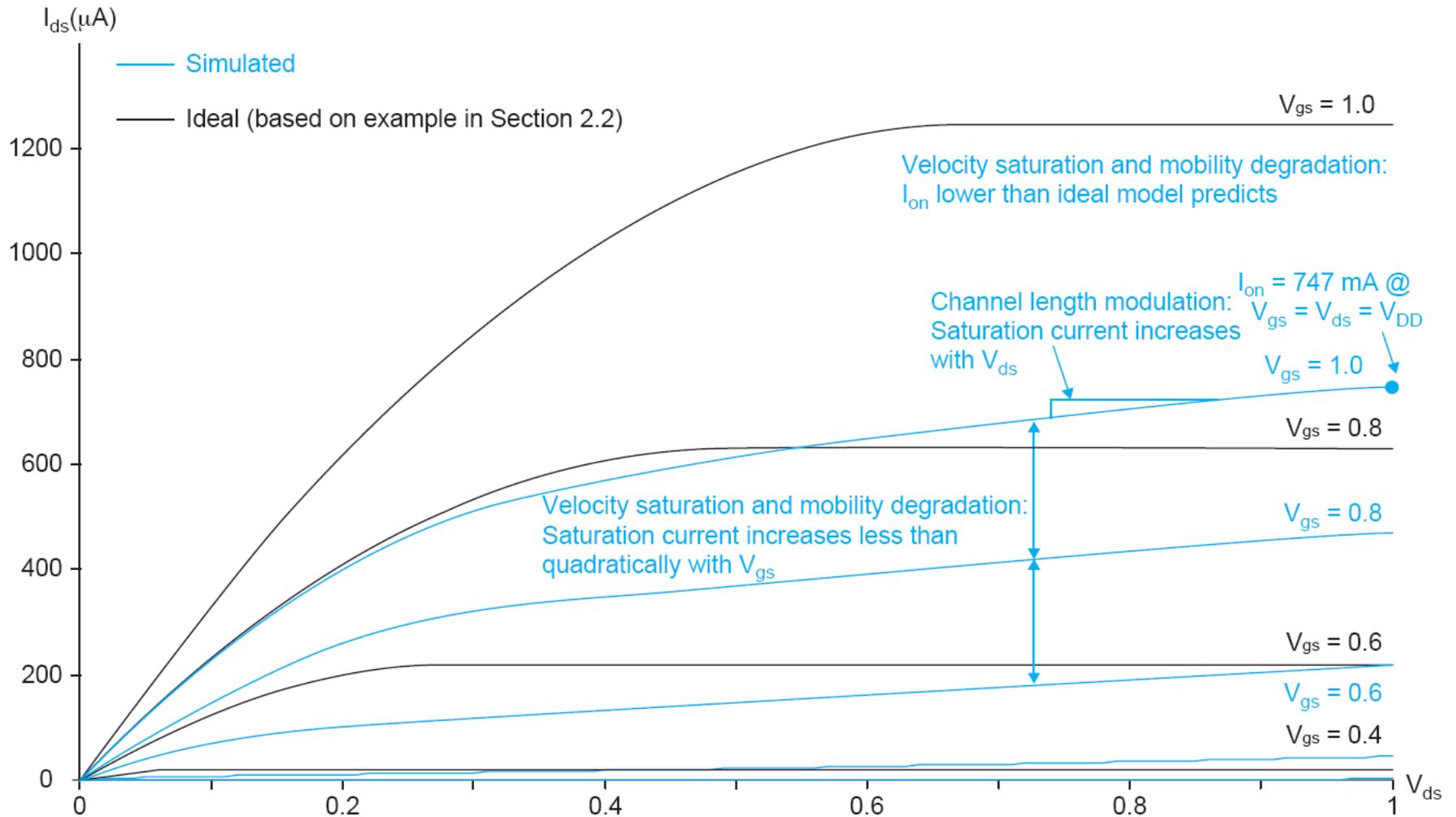
$$i_{ds} = -v_{out}/R$$

$$\text{voltage gain} = \frac{v_{out}}{v_{in}} = \frac{-g_m}{g_{ds} + 1/R}$$



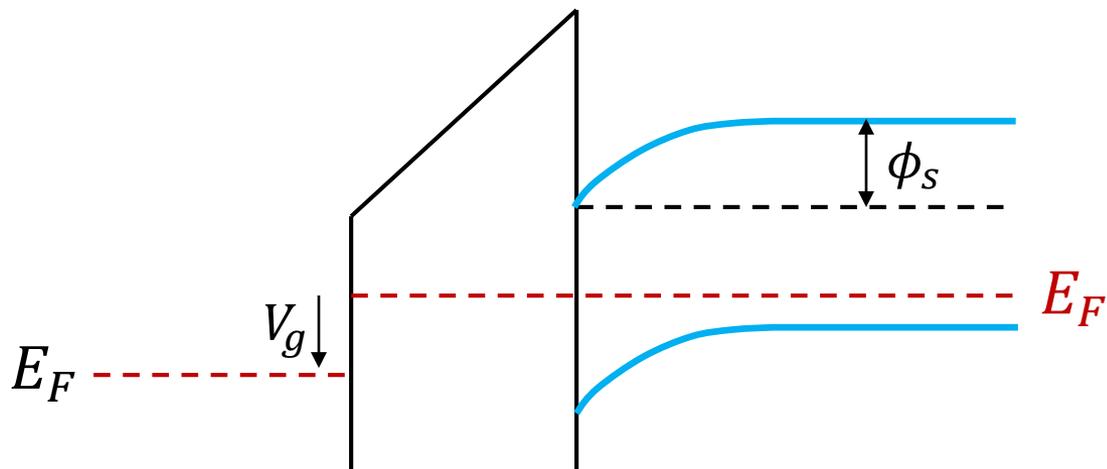
- A smaller g_{ds} is desirable for large voltage gain.
- Maximum available gain (or intrinsic voltage gain) is g_m/g_{ds}

Combined Short Channel Effects



Sub-threshold Conduction

- Until now, we have assumed $I_{ds} = 0$ for $V_{gs} < V_t$
- But a small positive V_{gs} will increase band bending and lower surface potential at silicon-oxide interface



$$\phi_s \approx \text{constant} + V_g/\eta \quad \text{where } \eta = 1 + C_{dep}/C_{oxe}$$

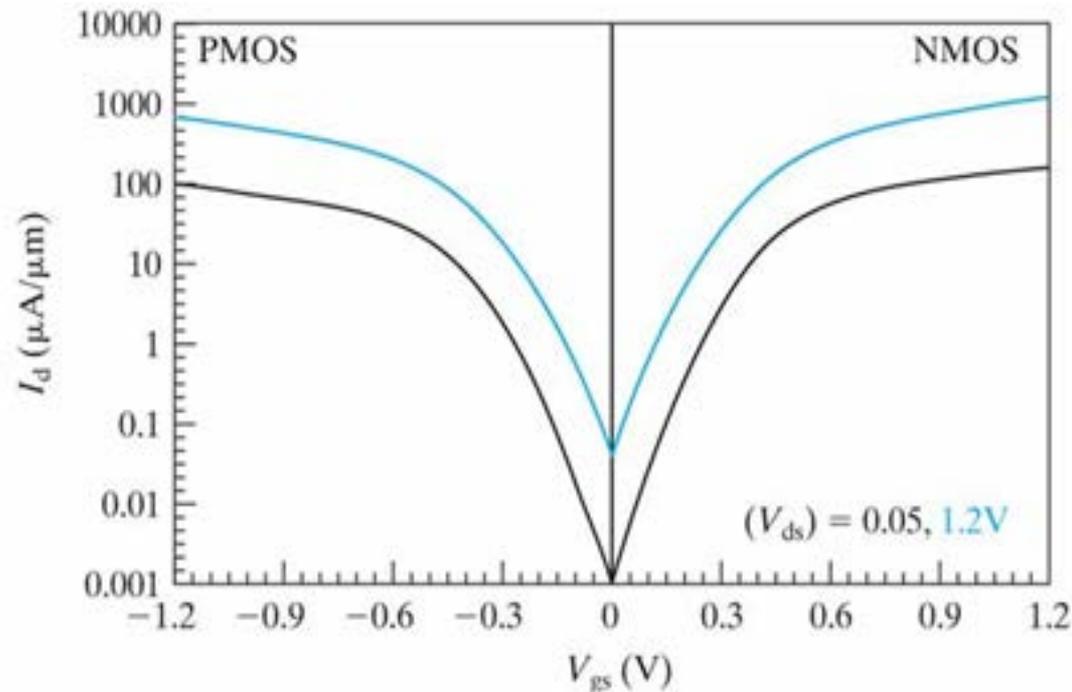
$$I_{ds} \propto n_s \propto e^{q \cdot \phi_s / kT}$$

$$I_{ds} \propto e^{q \cdot V_g / \eta kT}$$

in sub-threshold

Sub-threshold Current

- In sub-threshold, current increases exponentially with V_{gs}
- Current relatively independent of V_{ds} for $V_{ds} >$ a few kT
- In short channel length devices, more sensitivity to V_{ds} as V_t is reduced at high V_{ds} through a process known as drain-induced barrier lowering or DIBL



Sub-threshold Current

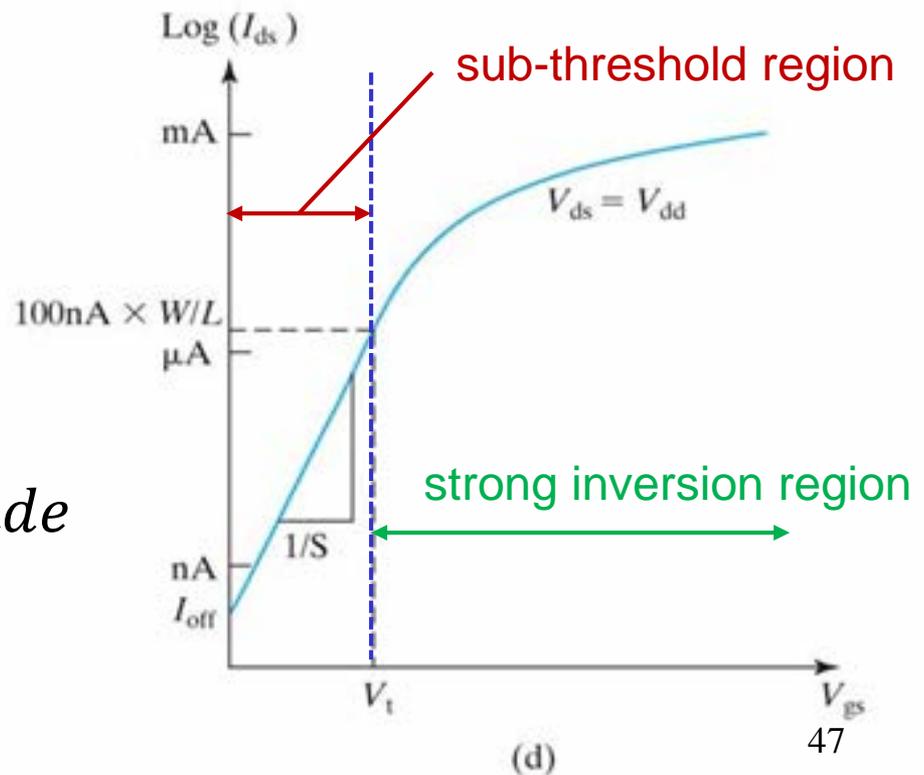
- Practical and common definition of V_t is the value of V_{gs} at which $I_{ds} = 100nA \times (W/L)$, then:

$$I_{ds}(nA) \approx 100 \cdot \frac{W}{L} \cdot e^{q(V_{gs}-V_t)/\eta kT} \quad \text{for } V_{gs} < V_t \text{ and } V_{ds} \gg kT$$

- Plotting $\ln(I_{ds})$ vs. V_{gs} :
 - Sub-threshold slope

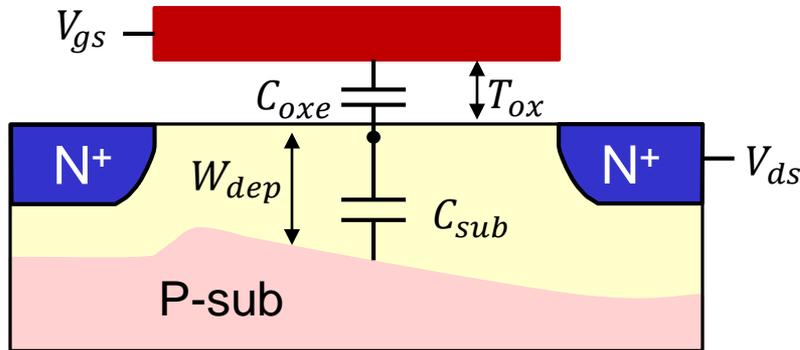
$$1/S = \eta kT / q$$

- S is typically 100 mV/decade at room temperature



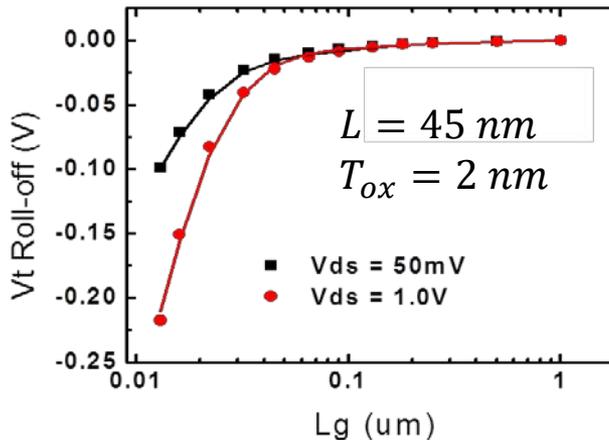
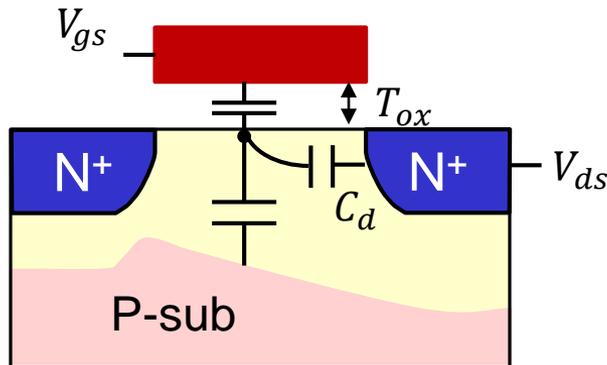
Drain Induced Barrier Lowering (DIBL)

- In a long channel device, surface potential is influenced by both gate potential and body potential



this is the back-gate bias effect – increases V_T

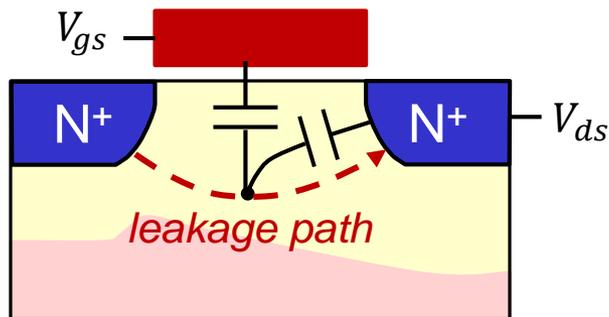
- In a short channel device, surface potential is also influenced by close proximity of drain



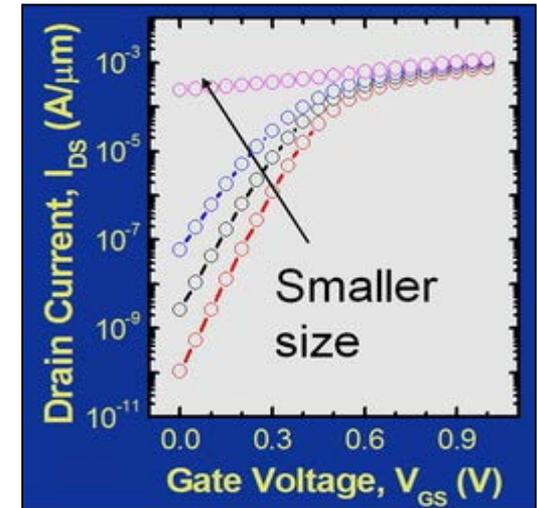
V_T is now a function of V_{ds} . Becomes harder to turn device off at high V_{ds} .

Sub-surface Leakage Paths

- Can reduce effect of drain on surface channel by increasing relative effect of gate
 - i.e. by increasing C_{ox} (reduce T_{ox} and/or increase ϵ_{ox})
- But there's another problem - leakage paths deep within transistor body:



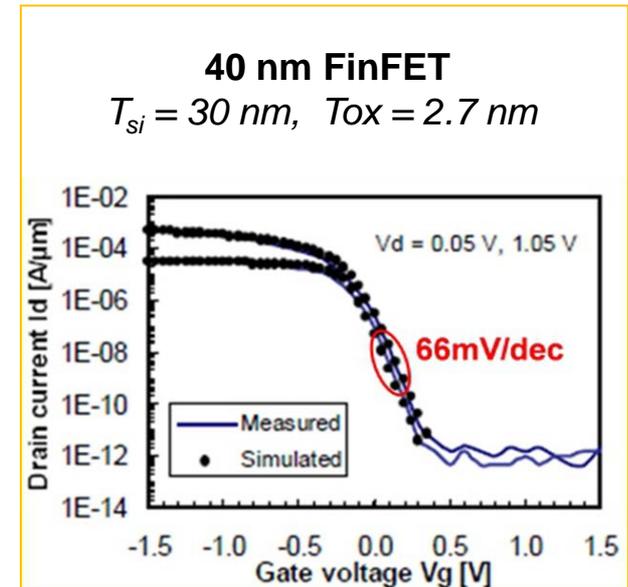
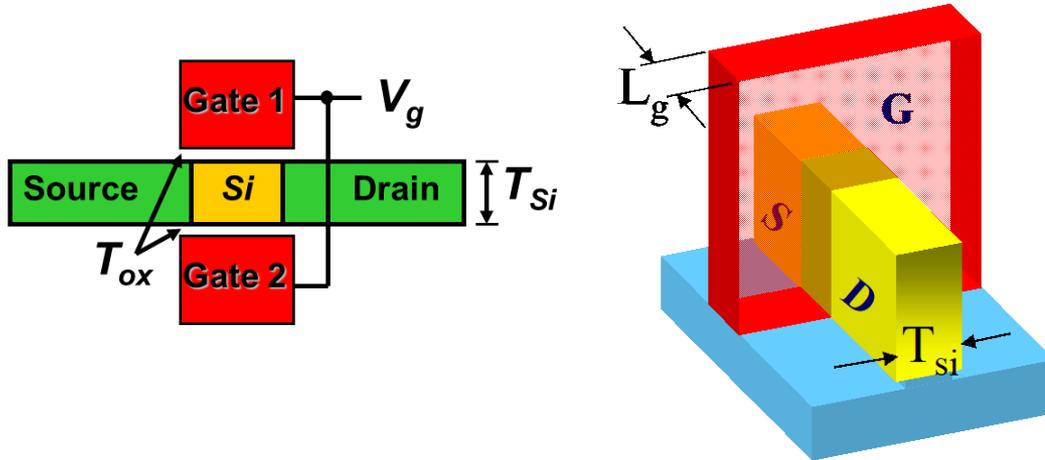
- Gate has much weaker control in this area
- Very difficult to turn device off



- Problem is regions in body that are as far from gate
 - allows drain to create a sub-threshold channel
- Need to completely re-think geometry of MOSFET

FinFET

- Maintain gate control by creating a narrow channel surrounded on all sides by the gate



- All portions of channel are very close to gate
 - gate maintains control of channel
 - good threshold control and sub-threshold slope
- Intel introduced FinFET into 22nm process (2012)