1 (15pts) Let $x, y, z \in P$, where $P$ is the set of positive integers. Let $Q(x, y, z) := x + y^2 - 2 \geq z$.

Are the following true or false? If false, explain.

1. $\forall x \forall y \exists z Q(x, y, z)$.

2. $\forall z \exists y Q(z, y, z)$.

3. $\forall x \exists y \forall z Q(x, y, z)$.

2 (15pts)

Let $A = \{x, y, z\}$, $B = \{1, 2, 3\}$ and $C = \{a, b\}$.

Find

1. $|P(A \times B \times C)|$.

2. $|A \times B|$.

3. A partition of $C^2 = C \times C$ into 2 sets.
Let $A, B, C$ be subsets of a universe $U$. If true, prove using the element method. If false, give a counterexample.

1. $(A \times C) \cup (B \times C) \subseteq (A \cap B) \times C$.

2. $A - (B \cup C) \subseteq (A - B) \cap (A - C)$. 
4 (14pts)

Let $m, n \in \mathbb{Z}$. Prove that if $m \cdot n$ is odd, then $m$ and $n$ must be odd.
Determine if the following argument is valid. Justify your answer with a truth table.

- $P_1$: If it is raining, then Bob is wet.
- $P_2$: If Bob is wet, then it is raining.
- $C$: Either it is raining or Bob is wet.
Let $\Sigma = \{a, b, c\}$ be an alphabet. Let $L_1$ be the language consisting of all strings over $\Sigma$ of length 3 in which the second symbol is an $a$ and let $L_2$ be the language consisting of all strings over $\Sigma$ of length 3 in which the second symbol is not a $b$. Find

1. $L_1 \cup L_2$

2. $L_1 \cap L_2$

3. $L_1 - L_2$
Prove by induction: \( \forall n \geq 1 \), if \( A \) is a set with \( n \) elements, then \( A \) has \( 2^n \) subsets.

*Remember:* Don’t forget to use induction.