1 (10pts) Let $x, y, z \in Q^+$, the positive rational numbers. Let $A(x, y, z) := x^2 + y^2 = z^2$.
Are the following true or false? If false, explain.

1. $\forall x \forall y \exists z A(x, y, z)$.

2. $\exists x \exists y \exists z A(x, y, z)$.

2 (12pts)
Let $A = \{a, b\}$, $B = \{1, 2, 3, 4\}$ and $C = \{x\}$. Find

1. $|A \times B \times C|$.

2. $|P(A \times B)|$.

3. $(C \times B) \times C$. 
Let $A, B, C$ be subsets of a universe $U$. If true, prove using the element method. If false, give a counterexample.

1. $A \times (B \cup C) \subseteq (A \times B) \cap (A \times C)$.
2. If $C \subseteq B - A$, then $A \cap B \neq \emptyset$.
3. If $A \subseteq B$, then $(A \cup C) \subseteq (B \cup C)$. 

4 (16pts)
Let $x \in \mathbb{Z}$. Prove (without using induction) that $x$ is odd if and only if $x^2 + 2$ is odd.
5 (14pts)
Determine if the following argument is valid. Justify your answer with a truth table.

- $P_1$: If we have a small bomb, then democracy is not safe.
- $P_2$: We do not have a small bomb.
- C: Therefore, democracy is safe.
Let $\Sigma = \{a, b, c\}$ be an alphabet. Let $L_1$ be the language consisting of all strings over $\Sigma^3$ which contain more $a$'s than $b$'s. Let $L_2$ be the language consisting of all palindromes over $\Sigma^3$, and let $L_3$ be the language consisting of all strings over $\Sigma^3$ having the same number of $b$'s and $c$'s. Find

1. $L_2$

2. $L_1 \cap L_3$

3. $L_3 - L_2$
7 (15pts)
Prove by induction: $\forall n \geq 1, \ 3 \mid (2^n - 1)$. 