1 (18pts)

- Let $\Sigma = \{a, b, c\}$ and let $f : \Sigma^* \rightarrow \Sigma^*$ be the function given by $f(w) = ac * w * ab$. Is $f$ 1-1, onto? If not, explain.

- Let $g : N \rightarrow N$ be the function given by $g(n) = n^6$. Is $f$ 1-1, onto? If not, explain.

- Let $h : Z \rightarrow Z$ be the function given by $h(n) = 5(n - 1)$. Is $h$ 1-1, onto? If not, explain.

2 (16pts) Let $X = \{a, b, c, d, e\}$, $Y = \{p, q\}$ and $Z = \{0, 1, 2\}$.

- How many relations are there on the set $X$?

- How many functions have domain $X$ and codomain $Z$?

- How many onto functions have domain $X$ and codomain $Y$?
3. (12pts)

Use the Pigeonhole Principle to show that if any 5 points with integer coordinates are chosen in the plane, then the line segment joining at least 2 of them will have a midpoint with integer coordinates.

*Hint:* Look at the parity of the integer coordinates.
4 (12pts) Let $A = \{x \in \mathbb{R} \mid 0 < x < 6\}$.

- Define: A set $X$ is a countably infinite set.

- Is $A$ a countably infinite set? **Prove or disprove.**

5 (10pts)
Let the functions $f, g : N \to N$ be given by $f(n) = n + 3$ and $g(n) = n^2$. Find

- $(g \circ f)(x)$

- $g^3(2)$
Find
Consider the following relations on \( A \). Are they reflexive, symmetric, antisymmetric or transitive? If they are, simply note this by putting the letter \( R, S, A \) or \( T \) next to the relation. If not, explain why not.

1. \( A = \{ x, y, z, w \} \). \( R_1 = \{ (w, w), (y, z), (z, w), (y, x), (x, w), (w, w), (w, x) \} \).

2. \( A = \mathbb{Z} \). \( R_2 : \{ (x, y) \mid x^2 + y^2 = 12 \} \).

3. \( A = \mathbb{R} \). \( R_3 : \{ (x, y) \mid |x| \leq 1 \text{ and } |y| \geq 1 \} \).

4. \( A = \Sigma^* \), where \( \Sigma = \{ 0, 1 \} \). \( R_4 : \{ (w_1, w_2) \mid l(w_1) = l(w_2 + 1) \} \).