1 (10pts)
Let $A = \mathbb{N}$ and $R$ be the equivalence relation defined on $A$ given by :

$$\forall x, y, \in A, \ xRy \iff \lfloor x/20 \rfloor = \lfloor y/20 \rfloor.$$ 

Find the distinct equivalence classes of $R$. 
Let $A = \{1, 2, 3, 4, 5, 6, 7, 8\}$. Find an equivalence relation $R$ on $A$ in which $[1] = \{1, 5\}$, $[2] = \{2, 6\}$, and $(3, 4), (3, 8) \in R$.

What is $[4]$ in your relation?
3. (20pts)

• Find, if possible, a simple connected graph with 34 edges in which all vertices have degree 7. If not possible, explain why not.

• Draw, if possible, a tree whose vertices have degrees

  1. (5,3,2,2,2,1,1,1,1)
     If not possible, explain why not.

  2. (5,3,3,3,2,1,1,1,1,1,1,1).
     If not possible, explain why not.

• Must every tree with 16 or more vertices have a vertex of degree two? Either prove or disprove with a counterexample.
4. (20pts)

1. Let \( A = \{a, b, c, d\} \) and consider the relations \( R \) and \( S \). Find

   - \( S^s \)
   
   - \( S^t \)
   
   - \( SR \)

2. Let \( S \) be the relation on \( \mathbb{Z} \) given by

   \[ S = \{(x, y) \mid x + y \text{ is odd}\}. \]

   Find \( S^2 \).
5 (20pts) Let \( A = \{1, 3, 9, 15, 27, 30, 45, 120\} \) and let \((A, R)\) be the poset given by the “divides” relation.

- Draw the Hasse diagram for \( R \).

- Find all maximal elements.

- Find all minimal elements.

- Partition the elements of \( A \) into the least number of sets, each of which is an antichain.
6 (15pts) Prove by induction: For all \( n \geq 1 \), if \( f \) is a 1–1 function from \( A \) to \( B \), with \( |A| = |B| = n \), then \( f \) is an onto function.

*Hint 1:* First outline the structure of the argument. Then worry about the details.

*Hint 2:* Let \( f(a) = b \) and consider the function \( f \) restricted to the set \( A \setminus \{a\} \).