1 (30pts) Consider the 3 functions below.

- $f_1 : \mathbb{R} - \{0\} \to \mathbb{R}^+$, where $f_1(x) = 1/x^2$.
- $f_2 : \mathbb{Z} \to \mathbb{Z}$, where $f_2(n) = 3n + 2$.
- $f_3 : \mathbb{Z}^+ \to \mathbb{Z}$, where $f_3(n) = 2^n - n^2$.

Answer the following.

1. Is $f_1$ 1-1, onto? Explain.
2. Is $f_2$ 1-1, onto? Explain.
4. Find $f_3 \circ f_2$, if possible.
5. Find $f_2 \circ f_3$, if possible.
6. Find $f_1(B)$, where $B = [-4, -1]$.
2 (20pts) Let $A = \{1, 2, 3, 4, 5\}$ and $B = \{x, y\}$.

1. How many different functions $f$ have domain $A$ and codomain $B$ with $f(3) = y$?

2. How many different onto functions have domain $A$ and codomain $B$?

3. How many different 1-1 functions have domain $A$ and codomain $B$?

4. Find all functions $g$ from $B$ to $B$ such that $g(x) = g^2(x)$.

5. How many functions in (4) have inverses?
3. (16pts) Let $T = [99]$. Use the Pigeonhole Principle to show that if 34 integers are selected from $T$, then the difference in absolute value between some 2 of them must be less than 3.
• Define: A set $X$ is a countably infinite set.

• Let $X$ be the set of all finite strings of $a$’s, $b$’s and $c$’s. Prove that $X$ is a countably infinite set.

• True or false: The set of all infinite strings of $a$’s, $b$’s and $c$’s form countably infinite set.
5 (18pts) Let $f : X \to Y$ and $g : Y \to Z$ be onto functions, where $X, Y$ and $Z$ are finite sets with $|X| = |Y| = |Z|$.

1. Prove $gof$ is an onto function.

2. Is $gof$ a 1 – 1 function? Explain.