1 (18pts) Let $A = \{a, b, c\}$ and $B = \{1, 2, 3, 4, 5\}$.

1. How many different relations are there on $A$?

2. How many of the relations in (1) are equivalence relations? Explain.

3. Find an equivalence relation on $B$ having 3 equivalence classes. Show the relation graphically.
Given the relations $R$ and $S$ on \{a, b, c, d\}, find

- $S^{-1}$
- $R^2$
- $S^t$
- $RS$
3 (24pts) Consider the Hasse diagram $H$ for the poset $(A, R)$ on $A = \{a, b, c, d, e, f, g, h, i, j, k, l, m\}$.

- Find all maximal elements.

- Find all minimal elements.

- Is $d \leq g$? Explain.

- Is $(A, R)$ a total order relation on $A$? Explain.

- Find a longest chain in $H$.

- Partition $A$ into the smallest possible number of antichains.
Prove by induction: $\forall n \geq 1, \ 6 \mid (n+7)(n+8)(n+9)$.
5 (12pts) Decide if each of the following relations on the given set $A$ is reflexive, symmetric, anti-symmetric and/or transitive. If not, explain.

1. $A = \{a, b, c, d, e\}$
   $$R_1 : \{(a, a), (b, a), (b, b), (b, d), (a, e), (a, d), (b, c), (c, d), (d, b), (d, c), (d, d), (d, a), (e, e)\}$$

2. $A = Z$
   $$R_2 : \{(x, y) \mid |x - y| \leq 1\}$$

3. $A = R$
   $$R_3 : \{(x, y) \mid |x| = |y|\}$$
6 (14pts) Prove that if $R$ is a transitive relation on a set $A$, then $R^{-1}$ is also transitive.