1 (15pts)

Use a truth table to check if the following is a tautology.

\[ [p \rightarrow (q \land r)] \iff [(p \rightarrow q) \land (p \rightarrow r)] \]
Let $A, B, C$ be sets. Prove using the element method or give a counterexample.

1. $A \times (B - C) \subseteq (A \times B) - (A \times C)$.
2. $C - (B - A) = (C - B) - A$.
3. $(A \cup B) \cap (A \cap B)^c \subseteq (A - B) \cup (B - A)$
3. (15pts)
Let $m, n \in \mathbb{Z}$. Prove the following: If $(m + n)^2$ is odd, then either $m$ is odd or $n$ is odd.
Let $U = \mathbb{Z}$. Write the negation of the following statements without using the negation symbol ($\neg$).

1. $\forall x \ \exists y \ [(x < y) \iff x^2 < y^2]$

2. $\forall x \forall y \ [(x < y) \rightarrow \exists z \ (x < z < y)]$
5 (15pts)
Let $\Sigma = \{0, 1\}$ be an alphabet and let $A = \{0, 11\}$ and $B = \{1, 10, 110\}$. Let $C$ be the set of all palindromes on $\Sigma^*$.

*Recall:* $A \star B$ is the set of all words obtained by concatenating a word from $A$ with a word from $B$.

Find

1. $B \star B$
2. $(A \star B) \cap (B \star A)$
3. $C \cap ((A \star A) \star A)$
Let $a \in \mathbb{R}$, $a > 1$. Prove by induction: $\forall n \geq 1, (a - 1) \mid (a^n - 1)$. 