1 (15pts)

Use a truth table to check if the following is a tautology.

\[ [q \rightarrow (p \land r)] \iff [(q \rightarrow p) \land (q \rightarrow r)] \]
2 (24pts)

Let \(A, B, C\) be sets. Prove using the element method or give a counterexample.

1. \(B \times (A - C) \subseteq (B \times A) - (B \times C)\).

2. \(C - (A - B) = (C - A) - B\).

3. \((A \cup C) \cap (A \cap C)^c \subseteq (A - C) \cup (C - A)\)
Let $m, n \in \mathbb{Z}$. Prove the following: If $(m + n)^2$ is even and $m$ is odd, then $n$ is odd.
4 (16pts)
Let $U = \mathbb{Z}$. Write the negation of the following statements without using the negation symbol ($\neg$).

1. $\forall y \exists x [(y < x) \iff y^2 < x^2]$

2. $\forall x \forall y [(x < y) \to \exists z (x < z < y)]$
5 (15pts)
Let $\Sigma = \{0, 1\}$ be an alphabet and let $A = \{0, 11\}$ and $B = \{1, 10, 100\}$. Let $C$ be the set of all palindromes on $\Sigma^*$.

Recall: $A \star B$ is the set of all words obtained by concatenating a word from $A$ with a word from $B$.

Find

1. $B \star B$
2. $(A \star B) \cap (B \star A)$
3. $C \cap ((A \star A) \star A)$
Let $x \in \mathbb{R}$, $x > 1$. Prove by induction: $\forall n \geq 1, \ (x - 1) \mid (x^n - 1)$. 