Let $X = \{x, y, z\}$, $Y = \{0, 1\}$, and $Z = \{q, r, s\}$.

1. How many relations are there from $X$ to $Z$?

2. How many functions are there from $X$ to $Z$?

3. How many onto functions are there from $X$ to $Y$?

4. How many onto functions are there from $X$ to $Z$?
2 (18pts) Consider the following functions.

\[
r : R \rightarrow R \\
x \rightarrow x^4
\]

\[
s : R \rightarrow R \\
x \rightarrow x^2 + 1
\]

\[
t : R \rightarrow R \\
x \rightarrow x + 2
\]

1. Find \( r(A) \), where \( A = [-1, 1] \)

2. Find \( r^{-1}(B) \), where \( B = (0, 3) \)

3. Solve \( (s \circ t)(x) = (t \circ s)(x) \).
3 (18pts) Are the following functions 1–1, onto? If not, explain why not.

\[ f : \mathbb{R} \rightarrow \mathbb{R} \]
\[ x \rightarrow x^2 + 1 \]

\[ g : \mathbb{R} \rightarrow \mathbb{R} \]
\[ x \rightarrow \frac{x^2 + 1}{x^2 + 2} \]

\[ h : \mathbb{R} - \{-2\} \rightarrow \mathbb{R} \]
\[ x \rightarrow \frac{x + 1}{x + 2} \]
4. (16pts) There are 51 houses on a street and each house has an address between 1000 and 1099, inclusive. Use the Pigeonhole Principle to show that at least 2 houses have addresses that are consecutive integers.
Let $A$ and $B$ be sets. Define

- $A$ is countably infinite
- $|B| = c$.

**Prove:** $\mathbb{Z}^+ \times \mathbb{Z}^+$ is a countably infinite set.

**Hint:** Use an argument “similar” to the argument used to show that $\mathbb{Q}^+$ is countably infinite.
6 (16pts) Consider the function $f : \mathbb{R} \rightarrow \mathbb{R}$ given by $f(x) = x + 3$. Prove

1. $f(x)$ is 1–1.

2. $f(x)$ is onto.