1 (16pts) Let $A = \{1, 2, 3, 4, 5\}$, $B = \{x, y, z\}$ and $C = \{a, b\}$.

1. How many relations are there from $A$ to $C$?

2. How many different functions $f$ have domain $A$ and codomain $B$ with $f(2) = x$?

3. How many different onto functions have domain $A$ and codomain $C$?

4. How many different 1-1 functions $g$ have domain $C$ and codomain $A$?
2. (14pts) Let $T = \{1, 3, 5, \ldots, 21, 23, 25\}$. Use the Pigeonhole Principle to find the integer $k$ such that

1. If $k$ integers are chosen from $T$, some 2 of them will sum to 26

    and

2. It is possible to select $k - 1$ integers from $T$ such that no two of them sum to 26.

Explain your answer.
3 (14pts) For each of the following functions \( f : A \rightarrow B \) determine if the function is 1-1. If not, explain. Also determine if the function is onto. If not, explain.

- \( A = \mathbb{Z}^+, \; B = \mathbb{R}, \; f_1(x) = x^{1/5} \).

- \( A = \Sigma^+, \; \Sigma = \{0, 1\}, \; B = \mathbb{Z}^+, \; f_2(w) = l(w). \)

  Note: \( l(w) \) is the length of \( w \).

Find the following.

1. \( f_1^2 \).

2. \( f_2^{-1}(A) \), where \( A = \{2, 3\} \).

3. \( f_1(B) \), where \( B = [1, 6] \).
• Define: A set $X$ is a countably infinite set.

• Let $A$ be the unit square in the plane joining the 4 points $(-1, 1), (1, 1), (1, -1)$, and $(-1, -1)$ and let $C$ be the unit circle \(\{(x, y) \mid x^2 + y^2 = 1\}\).

Outline an argument to show that $A$ and $C$ have the same cardinality.
1. Let \( f : X \rightarrow Y \) and \( g : Y \rightarrow Z \) be 1–1 functions. Prove that \( gof : X \rightarrow Z \) is a 1–1 function.

2. Let \( g : A \rightarrow B \), where \( A = R - \{-1\}, \: B = R, \: g(x) = \frac{x - 1}{x + 1} \).

   - Is \( g \) 1–1? Prove or disprove.

   - Is \( g \) onto? Prove or disprove.
Consider the following relations on $A$. Are they reflexive, symmetric, antisymmetric or transitive? If they are, simply note this by putting the letter $R$, $S$, $A$ or $T$ next to the relation. If not, explain why not.

1. $A = \{0, 1, 2, 3\}$. \(R_1 = \{(0, 0), (0, 1), (0, 2), (0, 3), (1, 1), (1, 2), (1, 3), (2, 1), (2, 3), (3, 3)\}$.

2. $A = \mathbb{Z}$. \(R_2 : \{(x, y) \mid \frac{y}{x} \in \mathbb{Z}\}$.

3. $A = \mathbb{Z}$. \(R_3 : \{(x, y) \mid x = |y|\}$.