1 (10pts)
Let $A = \{a, b, c\}$. Find all equivalence relations on $A$. **Explain** why these are the only equivalence relations.
2 (20pts) Let $A = \{1, 2, 3, 4, 5, 6, 7\}$. Find an equivalence relation $R$ on $A$ in which $(1, 3) \in R$ and $(5, 6) \notin R$.

What are the distinct equivalence classes in your relation?

*Hint:* Show the relation graphically.
3. (20pts)

1. Find two simple connected graphs, $G_1$ and $G_2$, that are not isomorphic and such that each has 8 vertices of degree 4.

   **Explain** why they are not isomorphic.

2. Draw, if possible, a tree whose vertices have degrees (3,3,2,2,1,1,1,1).

   If not possible, **explain why not**.

3. Draw, if possible, a tree whose vertices have degrees (4,2,2,2,2,1,1,1).

   If not possible, **explain why not**.
1. Let $A = \{a, b, c\}$ and consider the relations $R$ and $S$ below. Find

- $S^2$
- $S^{-1}$
- $RS$
- $R^t$
5 (16pts) Let $A = \{a, b, c, \ldots, i\}$ and let $(A, R)$ be the poset given by the Hasse diagram below.

- Find all longest chains.

- Partition $A$ into the smallest number of antichains.

- Find all maximal elements.

- Is $R$ a total ordering? Explain.
6 (14pts) Let $R$ be a relation on a set $A$. 

- Define $R^n$ for all $n \geq 0$.

- Prove by induction: For all nonnegative integers $a, b$, $R^{a+b} = R^a R^b$. Explain each step in the proof.

*Hint*: You may use the fact that the composition of relations is associative.