1 (12pts) Consider each of the following relations on the given set $A$. Is it reflexive, symmetric, antisymmetric, transitive? If it is any of these, simply note this by putting the corresponding letter(s) $R$, $S$, $A$, $T$ next to the relation. Then for each property that does not hold, explain why not.

1. $A = \{1, 2, 3, 4\}$. $R_1 = \{(1, 1), (1, 2), (1, 3), (1, 4), (3, 4), (2, 2), (2, 3), (4, 4), (4, 3)\}$.

2. $A = \mathbb{Z}$. $R_2 : \{(x, y) \mid x^2 = y^2 + 4\}$.

3. $A = \mathbb{R}$. $R_3 : \{(x, y) \mid |y| = x\}$.
Let $A = \{a, b, c, d\}$. Find 5 distinct equivalence relations on $A$ in which $a$ is related to $c$. For each relation

- What is $[b]$?
- Find the partition of $A$ associated with the relation.
3 (16pts) Consider the relations $R$ and $S$ below. Find

- $RS$
- $R^t$
- $R^2$
- $S^s$
Consider the relation $R$ of subset inclusion on the following sets in $A$:

$$A = \{\{2\}, \{3\}, \{2, 3\}, \{4, 5\}, \{2, 3, 5\}, \{3, 4, 5, 6\}, \{1, 2, 3, 5\}\}.$$

- Draw the Hasse diagram $H$ for the relation $R$.
- Find all maximal elements.
- Find all minimal elements.
- Find a longest chain in $H$.
- Partition the elements of $A$ into the smallest possible number of antichains.
5 (12pts) Let $R$ be a symmetric relation on a set $A$. Prove that $R^{-1}$ is symmetric.
Consider the 4 functions below.

- $f_1 : R - \{0\} \rightarrow R^+$, where $f_1(x) = 1/x^4$.
- $f_2 : Z \rightarrow Z$, where $f_2(n) = 3n + 5$.
- $f_3 : R \rightarrow R$, where $f_3(x) = e^x$.
- $f_4 : R - \{-1\} \rightarrow R$, where $f_4(x) = x/x + 1$.

Answer the following.

1. Is $f_1$ 1-1, onto? If not, explain.
2. Is $f_2$ 1-1, onto? If not, explain.
3. Is $f_3$ 1-1, onto? If not, explain.
4. Is $f_4$ 1-1, onto? If not, explain.
7 (12pts) Let $A = \{1, 2, 3, 4, 5\}$ and $B = \{x, y\}$.

1. How many different functions $f$ have domain $A$ and codomain $B$ with $f(4) = x$?

2. How many different onto functions have domain $A$ and codomain $B$?

3. How many different 1-1 functions have domain $A$ and codomain $B$?