1 (10pts) Let $f : A \rightarrow B$ and $g : B \rightarrow C$ be injective (1-1) functions. Prove that $gof : A \rightarrow C$ is an injective (1-1) function.

*Hint:* Get the first step right and the last step right!
2 (24pts)

Are the following functions 1-1? onto? If not, explain.

1. $f : N \rightarrow \mathbb{N} \times \mathbb{N}$
   
   $n \rightarrow (n, n)$

2. $g : \mathbb{N} \times \mathbb{N} \rightarrow \mathbb{N}$
   
   $(m, n) \rightarrow 2m + n$

3. $h : N \rightarrow N$
   
   $n \rightarrow n^2$

- Find $(h \circ g) \circ f$.
- Find $h \circ (g \circ f)$. 
3. (16pts) Let $\Sigma = \{a, b, c, d, e\}$ and let $W$ be the set of finite words over $\Sigma$ of length at least two. Use the Pigeonhole Principle to find the size of a smallest set of words in $W$ with the property that some 2 words will begin and end with the same letters.

*Example:* abd and aecd begin and end with the same letters.

Show how you are using the Pigeonhole Principle.
1. Define what it means for a set to be **countably infinite**.

2. Show that \( Z^+ \times Z^+ \) is a countably infinite set.

   *Hint*: Recall that we proved \( Q^+ \) is countably infinite. Try to use a similar argument.
1. Define: \( T \) is a tree.

2. Prove that \( T \) must have a vertex of degree 1.

*Hint:* Consider a longest path \( P \) in \( T \) that begins at vertex \( u \) and ends at vertex \( v \).
Let $G$ be a connected planar graph having $n$ vertices, $e$ edges and $r$ regions.

Prove by induction:

$$n - e + r = 2.$$ 

*Hint:* Use induction on $e$. 