I pledge my honor that I have abided by the Stevens Honor System.

1 (14pts)

1. Find a function $f : \mathbb{R} \rightarrow \mathbb{R}$ that is $1 - 1$, but not onto.

2. Find a function $g : \mathbb{R} \rightarrow \mathbb{R}$ that is onto, but not $1 - 1$.

Hint: You may use pictures.
2 (14pts) Let $A, B, C$ be sets and $f : A \to B$ and $g : B \to C$ be onto functions.

1. **Prove** that $gof : A \to C$ is an onto function.

2. Is $gof : A \to C$ a 1–1 function? **Prove or give a counter-example.**
3. (16pts) Let $T = [24]$, i.e., $T = \{1, 2, 3, \ldots, 24\}$. Use the Pigeonhole Principle to find the unique integer $k$ such that

1. if $k$ integers are chosen from $T$, some 2 of them must sum to 25

and

2. it is possible to select $k - 1$ integers from $T$ such that no two of them sum to 25.

Explain, i.e., prove 1 and give an example for 2.
4 (16pts)

- Define: A set $X$ is a countably infinite set.

- Let $T$ be the set of all finite strings of 0’s and 1’s. Prove that $T$ is a countably infinite set.
Find, if possible, a connected graph on 9 vertices with the following properties.

1. vertex degrees $(4, 3, 3, 3, 2, 2, 2, 2)$ and not hamiltonian.

2. vertex degrees $(4, 4, 4, 3, 3, 3, 3, 2, 2)$ and eulerian.

3. a connected **plane** graph that has exactly 18 edges. Clearly number the edges.
6 (16pts) Prove by **induction**: \( \forall n \geq 1, \) the complete bipartite graph on \( 2n \) vertices, \( K_{n,n} \), has \( n^2 \) edges.