1 (18pts)

- Let $\Sigma = \{a, b, c\}$ and let $f : \Sigma^* \to \Sigma^*$ be the function given by $f(w) = ac * w * ab$. Is $f$ 1-1, onto? If not, explain.

- Let $g : N \to N$ be the function given by $g(n) = n^4$. Is $f$ 1-1, onto? If not, explain.

- Let $h : Z \to Z$ be the function given by $h(n) = 3(n - 1)$. Is $h$ 1-1, onto? If not, explain.
2 (16pts) Let \( X = \{a, b, c, d, e\} \), \( Y = \{p, q, r\} \) and \( Z = \{0,1\} \).

- How many relations are there on the set \( X \)?

- How many functions have \textbf{domain} \( X \) and \textbf{codomain} \( Y \)?

- How many \textbf{onto} functions have \textbf{domain} \( X \) and \textbf{codomain} \( Z \)?

- How many \textbf{1-1} functions \( g \) have \textbf{domain} \( X \) and \textbf{codomain} \( X \) with \( g(a) = b \)?
3. (15pts)
Use the Pigeonhole Principle to show that if any 5 points with integer coordinates are chosen in the plane, then the line segment joining at some 2 of them will have a midpoint with integer coordinates.

*Hint*: Look at the parity of the integer coordinates.
4 (12pts) Let \( A = \{ x \in \mathbb{R} \mid 0 < x < 4 \} \).

- Define: A set \( X \) is a countably infinite set.

- Is \( A \) a countably infinite set? **Prove or disprove.**
• Find a set $A$ and a $1 - 1$ function $f : A \rightarrow A$ that is not an onto function.

• Let $f : A \rightarrow B$ and $g : B \rightarrow C$ be onto functions. Prove that $gof : A \rightarrow C$ is an onto function.  

  \textit{Hint:} Get the first step right and the last step right!
6 (24pts) Find
Consider the following relations on $A$. Are they reflexive, symmetric, antisymmetric or transitive? If they are, simply note this by putting the letter $R, S, A$ or $T$ next to the relation. If not, explain why not.

1. $A = \{x, y, z, w\}$. $R_1 = \{(w, w), (y, z), (z, w), (y, x), (x, w), (w, w), (w, x)\}$.

2. $A = \mathbb{Z}$. $R_2 : \{(x, y) \mid x^2 + y^2 = 10\}$.

3. $A = \mathbb{R}$. $R_3 : \{(x, y) \mid |x| \leq 1 \text{ and } |y| \geq 1\}$.