1 (16pts)

1. Let $\Sigma = \{a, b, c\}$ and let $f: \Sigma^* \to \Sigma^*$ be the function given by $f(w) = bw$. Is $f$ 1-1, onto? If not, explain.

2. Find, if possible, a 1-1 function $f: \mathbb{Z} \to \mathbb{Z}$ that is not onto. If not possible, explain.

3. Find, if possible, an onto function $g: \mathbb{Z} \to \mathbb{Z}$ that is not 1-1. If not possible, explain.

4. Find, if possible, an onto function $h: A \to A$ that is not 1-1, where $A = [10]$. If not possible, explain.
2 (12pts) Let $X = \{p, q, r, s, t\}$, $Y = \{t, u\}$ and $Z = \{a, b, c\}$.

- How many relations are there on the set $X$?

- How many functions have domain $X$ and codomain $Z$?

- How many onto functions have domain $Z$ and codomain $Y$?

- How many 1-1 functions have domain $Z$ and codomain $X$?
3. (16pts)
Let $\Sigma = \{x, y, z\}$ be an alphabet. Let $L_1$ be the language consisting of all words over $\Sigma$ of length 3 in which the third symbol is $y$ and let $L_2$ be the language consisting of all words over $\Sigma$ of length 3 in which the second symbol is $x$. Find

1. $L_1 \cup L_2$
2. $L_1 \cap L_2$
3. $L_1 - L_2$
• Define: A set $X$ is a countably infinite set.

• Is $Z$ a countably infinite set? Prove or disprove.
Let $T = [19]$. Use the **Pigeonhole Principle** to show that if any 12 distinct integers are chosen from $T$, some 2 of them must sum to 20?
Let the functions $f, g : N \to N$ be given by $f(n) = n + 1$ and $g(n) = n^3$. Find

- $(g \circ f)(n)$

- $f^2(2)$

- $f(A)$, where $A = \{1, 2\}$

- $g^{-1}(B)$, where $B = \{1, 27\}$. 
Let \( f : A \to B \) and \( g : B \to C \) be injective (1-1) functions. Prove that \( g \circ f : A \to C \) is an injective (1-1) function.

*Hint:* Get the first step right and the last step right!