1 (15pts)

1. Let $U = N = \{0, 1, 2, \ldots\}$. Are the following true or false? Explain.
   - $\exists y \forall x \ [x + y = x]$.
   - $\forall x \exists y \ [xy = x]$.

2. Let $B = \{3, a, 4, c, 1, b, 2\}$. Is $\{a, b, 3\}, \{1, c\}, \{2, 4, a\}$ a partition of $B$? Explain.

2 (15pts)

Let $A = \{a, b\}$.

Find the power set $P(A)$.

True or false?

- $A \in A$?
- $A \subseteq P(A)$?
- $\{a\} \subseteq A$?
- $\{a\} \in A$?
3 (20pts)
Let $A, B, C$ sets. Prove using the element method or give a counterexample.

1. $(A \cup B) \cap C \subseteq A \cup (B \cap C)$.

2. $A \cup (B \cap C) \subseteq (A \cup B) \cap (A \cup C)$. 

4. (15pts)
Let $x$ and $y$ be integers. Prove that if $z = x + y$ is an odd integer, then either $x$ is odd or $y$ is odd.

*Hint:* Use the contrapositive.
Determine if the following argument is valid. Justify your answer with a truth table.

- \( P_1 \): If it is not snowing, then it is hot.
- \( P_2 \): It is snowing.
- \( C \): Therefore, it is not hot.

6 (20pts)
Let \( \Sigma = \{0, 1\} \) be an alphabet. Let \( L_1 \) be the language consisting of all strings over \( \Sigma \) of length 4 in which the first symbol is a 0 and let \( L_2 \) be the language consisting of all strings over \( \Sigma \) of length 4 in which the last symbol is a 1. Find

- \( L_1 - L_2 \)
- \( L_1 \cap L_2 \)