1 (15pts)
Let $\Sigma = \{a, b\}$ and $A = \Sigma^*$. Define a relation $R$ on the set $A$ as follows:

$$\forall s, t \in A, \ sRt \text{ iff } l(s) = l(t), \text{ i.e., the length of } s \text{ equals the length of } t.$$ 

- Show that $R$ is an equivalence relation.
- Find all distinct equivalence classes of $R$. 
Let $A = \{1, 2, 3, 4, 5, x, y, z\}$. Define an equivalence relation $R$ on the set $A$ in which $2 \sim 5$. What is $[x]$ in your relation?
Let \( A = \{1, 2, 3, 4\} \) and consider the relations \( R \) and \( S \) below. Find

- the symmetric closure \( R^s \).
- the transitive closure \( R^t \).
- \( RS \)
- \( S^2 \)
- \( S^{-1} \)
4 (20pts) Let $A = \{2, 3, 4, 6, 8, 9, 10, 12, 18, 24, 27, 30\}$ and let $R$ be the “divides” relation, i.e., $xRy$ iff $x$ divides into $y$ with no remainder.

- Draw the Hasse diagram for $R$
- Find all maximal elements
- Find all minimal elements
- Find a longest chain in $R$. 
5. (15pts) Prove by induction: For all integers $n \geq 1$,

$$\sum_{i=1}^{n} 2i^2 = \frac{n(n+1)(2n+1)}{3}.$$
6 (15pts) Prove by induction: For all integers $n \geq 1$,

$$\sum_{i=1}^{n} i^2(i - 1)! = (n + 1)! - 1.$$

Note: $0! = 1$