

Giancoli Ch 39-14: Free electron wave function.

Define useful constants.

$$\begin{aligned} \text{amu} &\equiv 1.66054 \cdot 10^{-27} \cdot \text{kg} & \text{nm} &\equiv 10^{-9} \cdot \text{m} & \hbar &\equiv 1.054573 \cdot 10^{-34} \cdot \text{joule} \cdot \text{sec} \\ k_{\text{B}} &\equiv 1.380658 \cdot 10^{-23} \cdot \frac{\text{joule}}{\text{K}} & c &\equiv 2.99792458 \cdot 10^8 \cdot \frac{\text{m}}{\text{sec}} & h &\equiv 2 \cdot \hbar \\ m_{\text{e}} &\equiv 9.11 \cdot 10^{-31} \cdot \text{kg} & N_{\text{A}} &\equiv 6.022137 \cdot 10^{23} \cdot \frac{1}{\text{mole}} & q &\equiv 1.602177 \cdot 10^{-19} \cdot \text{coul} \\ \epsilon_0 &\equiv 8.854188 \cdot 10^{-12} \cdot \frac{\text{farad}}{\text{m}} & \text{eV} &\equiv q \cdot \text{volt} & \text{kJ} &\equiv 10^3 \cdot \text{joule} \\ m_{\text{n}} &\equiv 1.008665 \cdot \text{amu} & \mu\text{m} &\equiv 10^{-6} \cdot \text{m} & & \end{aligned}$$

A free electron has a wave function given by $\psi(x) = A \sin(1.0 \times 10^{10} x)$ with x in meters. Find wavelength, momentum, speed, energy.

The general form for the wavefunction of a free particle is given by $\psi(x) = A \sin(kx)$ where k is the wave vector. From inspection we conclude:

$$k := 1 \cdot 10^{10} \cdot \text{m}^{-1} \quad \text{Note that with } x \text{ in meters, } k \text{ is in inverse meters.}$$

See the various equations in the chapter to verify the following assignments.

$$\lambda := \frac{2\pi}{k}$$

$$= 0.628 \cdot \text{nm}$$

$$p := \hbar \cdot k$$

$$p = 1.055 \cdot 10^{-24} \cdot \text{kg} \cdot \text{m} \cdot \text{sec}^{-1}$$

$$v := \frac{p}{m_{\text{e}}}$$

$$v = 1.158 \cdot 10^6 \cdot \text{m} \cdot \text{sec}^{-1}$$

$$\text{KE} := \frac{p^2}{2 \cdot m_{\text{e}}}$$

$$\text{KE} = 6.104 \cdot 10^{-19} \cdot \text{joule}$$

$$\text{KE} = 3.81 \cdot \text{eV}$$