ME345 Modeling and Simulation, Spring 2012  
Case Study 3  
Assigned: Friday April 20, 2012

Due Date 1 (for email confirmation of final grade): Wednesday May 9 at 5 pm  
Due Date 2 (absolute latest possible submission): Monday May 14 at NOON

Note 1: Assignment can either be submitted via hardcopy under my door (E-307) or emailed as a PDF document to Frank.Fisher@stevens.edu.

Note 2: There are three separate problems for Case Study 3, evenly weighted for the overall Case Study 3 grade.

Note 3: Each of the Case Studies should be done in groups of 4 students. Each student is expected to contribute equally to the group work. No two students can be in the same group for more than one Case Study (as discussed in class). Please complete an anonymous Group Member evaluation form and email it to the instructor after submission of the Case Study report.

Problem 1.

As shown in Figure 1, an $l = 45$ inch long hinged gate for an amusement park ride is comprised of a bar of mass $m$ with a concentrated end mass $M$. The gate rotates about point $O$ and is connected to a torsional spring ($k_t$) and a torsional damper ($b_t$), which result in forces proportional to $\theta$ and $\dot{\theta}$, respectively (with $\theta$ defined as in Figure 1). The behavior of the system is described by a second order differential equation (note analogy with a linear spring-mass-damper system) as:

$$J_0 \ddot{\theta} + b_t \dot{\theta} + k_t \theta = 0$$

where $J_0$ is the polar moment of inertia of a hinged beam, which for a beam with a concentrated tip mass can be written as

$$J_0 = \frac{1}{3} (m + 4M)l^2$$

Part 1. Find values for the torsional spring ($k_t$) and torsional damper ($b_t$) with which the gate to closes to within 5° of the equilibrium position (i.e. horizontal) within 2 seconds after being opened to $\theta = 75^\circ$ (hint: this is one your initial conditions; what is the other?). Assume initially that the concentrated mass is such that $M = 1$ m.

Part 2. When designing the gate, not only does the designer want it to ‘close’ quickly, but there is the desire to not make it so stiff that it cannot be opened. In this case, the person entering the gate applied a force $F$ (assumed to be at $l = 45^\circ$) to create a moment on the system.

In this case, the differential equation can now be written in the form

$$J_0 \ddot{\theta} + b_t \dot{\theta} + k_t \theta = M(t)$$
where $M(t)$ is the moment applied by the park-goer. (In this case as a first approximation we can assume that the moment is constant as a function of time – ignoring the change in length of the moment arm as the gate swings. If you wish, you can include the change in length of the moment arm in your later calculations as well.)

![Figure 1. Schematic of an amusement park gate.](image)

What is the force that the park-goer needs to apply to open the gate that you designed in Part 1, and is it reasonable? If not, how can you iterate the design to provide an operable gate design?

Report for Part 1:

You will need to submit an engineering report of your analysis to the company president (i.e. the professor). The report should be well-organized, clear, and concise, and at minimum address the points listed below. The report should also provide a justification of why the results that you have obtained are sensible. **NOTE: just submitting the software output without your analysis and discussion is NOT acceptable.**

While it is natural to discuss your work and progress with your colleagues (i.e. classmates), **individual group analyzes and reports are required** [this cannot be emphasized enough – work that fails to meet this requirement will not be given credit for the assignment]. Your supervisor (i.e. the TA) is also available to answer thoughtful questions as you work on the project, but it would be unprofessional to overly rely on your supervisor to complete your project.

- Describe briefly the specifics of the creation of your model in Simulink. In particular mention any aspects that you feel might be ‘noteworthy’ or unusual.

- Summarize your results. Be sure to compare the results of your model to a simplified ‘by hand’ calculation(s) to justify your results. **BE SURE TO USE RADIANS IN YOUR ANALYSIS**

- Discuss how you found appropriate values for the torsional spring ($k_t$) and torsional damper ($b_t$) with which the gate to closes to within 5° in 2 seconds. Show plots of the behavior of the gate as designed.

- Given your solution to Part 1 of the project, discuss how you determined the force that the park-goer would need to apply to open the gate. Are these values appropriate, or does the gate need to be redesigned?

- How would you optimize the behavior of the gate so that both ‘ease of opening’ and ‘quickness of close’ are optimal? Do you think you have such a solution? Why or why not?

- Are their any recommendations to your supervisor regarding ‘next steps’ in the analysis?
Problem 2.

A one degree of freedom model of a vehicle traveling over a rough road is shown in Figure 2. The vehicle has a mass of 1200 kg. The suspension has a spring constant $k = 400 \text{kN/m}$ and a damping constant $c = 20 \times 10^3 \text{kg/s}$. Initially assume a constant vehicular velocity $v = 100 \text{ km/hr}$, traveling on a road surface modeled as sinusoidal with an amplitude of $Y = 0.05 \text{ m}$ and a wavelength of $\lambda = 6 \text{ m}$.

\[ m\ddot{x} + c(\dot{x} - \dot{y}) + k(x - y) = 0 \]

\[ \omega = 2\pi f = 2\pi \frac{v}{\lambda} \]

Using Simulink, analyze the system and address (at minimum)

a) (Before using Simulink). Derive/show that the appropriate state equation for the system, neglecting the constant gravitational force $mg$, can be written:

\[ m\ddot{x} + c(\dot{x} - \dot{y}) + k(x - y) = 0 \]

b) How do the mass, spring stiffness, and damping constant effect the system behavior for the vehicle speed and road conditions given above?

c) Using the system parameters given initially in the problem, discuss how the amplitude of the road bumpiness and speed of the car affect the system behavior.

d) Discuss the simulation results in relation to the analytical solution for a single DOF base excitation vibration problem, where:

\[ \frac{X}{Y} = \left\{ \frac{1 + (2\zeta r)^2}{(1-r^2)^2 + (2\zeta r)^2} \right\}^{1/2} \]

$X =$ the displacement amplitude due to the vibration

$Y =$ the amplitude of the base vibration (0.05m for the original problem parameters)
\[ r = \text{the frequency ratio} \ (r = \frac{\omega}{\omega_n}) \]
\[ \omega_n = \text{the natural frequency} = \sqrt{\frac{k}{m}} \]
\[ \zeta = \text{the damping ratio} \ (\zeta = \frac{c}{c_c}) \]
\[ c_c = \text{the critical damping factor} \ (c_c = 2m\omega_n) \]

(As a check, for original problem parameters I find displacement amplitude \( X \) is \( \approx 0.0417 \) m)

e) Extend the model to account for the dynamics behavior of the car going over a pothole and/or an isolated speed bump. Describe in detail how you have attempted to model the problem, how you choose to analyze the results, and whether you feel that your model captures the essence of the behavior of the system.

Report:

You will need to submit an engineering report of your analysis to the company president (i.e. the professor). The report should be well-organized, clear, and concise, and at minimum address the points listed above. The report should also provide a justification of why the results that you have obtained are sensible. **NOTE: just submitting the software output without your analysis and discussion is NOT acceptable.**

While it is natural to discuss your work and progress with your colleagues (i.e. classmates), **individual group analyzes and reports are required** [this cannot be emphasized enough – work that fails to meet this requirement will not be given credit for the assignment]. Your supervisor (i.e. the TA) is also available to answer thoughtful questions as you work on the project, but it would be unprofessional to overly rely on your supervisor to complete your project.
Problem 3.

FTF Enterprises makes automotive parts, in particular, Camshafts & Gears. The Unit Profit for Camshafts is $25/unit, while the Unit Profit for Gears is $18/unit. To make these products FTF Enterprises needs Steel, Labor, and Machine Time. In total, 5000 lbs of steel is available, 1500 hours of labor are available, and 1000 hours of machine time are available. Camshafts require 5 lbs of steel, 1 hour of labor, and 3 hours machine time, while Gears need 8 lbs of steel per part, 4 hours of labor, and 2 hours machine time.

Part A:

1. Clearly identify your optimization function and list your constraints. What are your design parameters (i.e. what can you change to optimize the problem)?

2. Prepare an analysis in Excel using the optimization/solver tool. (Hint: to check your model, if FTF were to produce 100 camshafts and 100 gears, the total profit would be $4300 with 1300 pounds of steel used.)

3. How many camshafts & gears would you recommend that the company make in order to maximize profit? What is the maximum profit in this case? What parameters are limiting the profits that the company can obtain?

Part B: Due to a shortage of steel, the maximum amount of steel that can be obtained is 2500 pounds. How does this affect the optimal manufacturing numbers?

Part C: The analysis above does not include other relevant costs (for example, the cost of the raw materials) into the calculations. By qualitatively analyzing the results from the original problem (5000 lbs of steel available) and the new problem (2500 lbs of steel available), can you make suggestions to the owners of FTF Enterprises on how they may be able to increase their profits? (Hint: how do the profits change for the case when the amount of available steel decreases?)

In a SHORT report for this third part of the Case Study, provide a SHORT (one page max) description of your work, plus any relevant Excel screen shots.

The ‘writeup’ for the third problem should be in summary form, whereas the reports for the first two problems should be in a more standard form. The Report should be a single file with three ‘chapters’ representing each of the assigned problems.