ME345 Modeling and Simulation, Spring 2018
Case Study 3
Assigned: Friday April 20

Due Date 1 (for email confirmation of final grade): Thursday May 10 (11:59 pm)

Due Date 2 (absolute latest possible submission): Monday May at 3pm (no submissions will be accepted after this time)

Note 1: Each of the Case Studies will be done in groups of up to 5 students. Each student is expected to contribute equally to the group work. No two students can sign up to be in the same group for more than one Case Study (as discussed in class).

Note 2: Students should complete and upload an anonymous Group Member evaluation immediately after the Case Study assignment for their group has been uploaded.

Note 3: There are three separate problems for Case Study 3; note that they are not equally weighted. **The Report should be a single file with three 'chapters' representing each of the assigned problems.** One report per group should be submitted through CANVAS by one of the deadlines indicated above.

Note 4: All team members are responsible for all aspects of the project work.

Problem 1 (35% of grade)

In this problem you will implement the finite element method by hand to determine solve the problem of a *beam* subjected to applied external load.[1] The *element stiffness matrix* for the generic beam element shown in Figure 1 can be written as
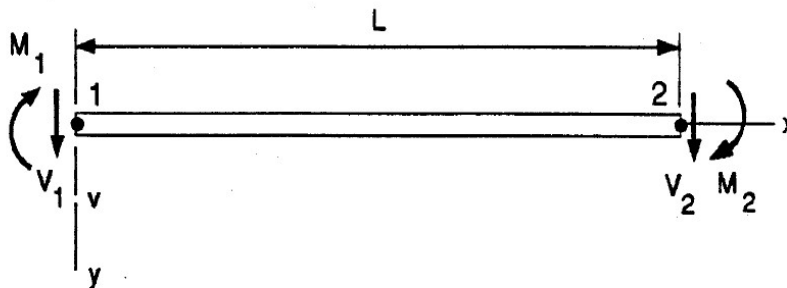

Figure 1. Generic beam element

$$\begin{bmatrix} V_1 \\ M_1 \\ V_2 \\ M_2 \end{bmatrix} = \frac{EI}{L^3} \begin{bmatrix} 12 & 6L & -12 & 6L \\ 6L & 4L^2 & -6L & 2L^2 \\ -12 & -6L & 12 & -6L \\ 6L & 2L^2 & -6L & 4L^2 \end{bmatrix} \begin{bmatrix} v_1 \\ \theta_1 \\ v_2 \\ \theta_2 \end{bmatrix}$$

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[1] The solution to this problem is analogous to earlier problems that we have done using the finite element approach, except here that we are using an element with two degrees of freedom per node.

where E is the modulus, I is the moment of inertia, and L is the length of the element, while V_i and M_i are the externally applied shear forces and moments and v_i and θ_i are the vertical displacements and angles of twist of each node as shown in the figure, respectively.
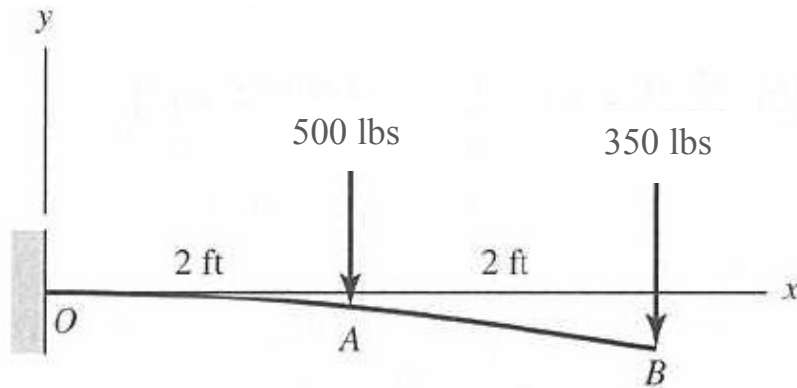

Figure 2. Cantilevered beam subject to two point (shear) loads. The modulus of the beam is 30 Msi and the moment of inertia of each element is 15 in⁴. Note that in this problem there are no applied external moments ($M_A = M_B = 0$)

Part A. For the problem shown in Figure 2, apply the definition of the element stiffness matrix to solve for the displacements at points A and B. As you go through the problem, be sure to *clearly* identify:

1. the element stiffness matrix of each element specific to the details of the problem

2. the global stiffness matrix (which will be a 6 x 6 matrix)

3. the 'simplified' global stiffness matrix, which is reduced to a 4 x 4 matrix due to the application of zero-value boundary conditions

4. the vertical displacements of nodes A and B

Part B. Check the solution to your problem using an independent verification, such as: 1) a computer-based approach, or 2) an alternative hand-based calculation such as superposition using standard tables such as those found in your favorite edition of *Shigley's Mechanical Engineering Design*. (Note that ONE alternative check will be considered a minimum; TWO or more independent verifications will be necessary to receive maximum points for this Problem.) When checking your result it may be necessary to carefully select a cross-sectional geometry which results in the proper value for the moment of inertia in Part A.

Report for Problem 1.

In a concise report for this part of the Case Study, provide a clear and legible solution to the problem in Part A. Also show the results for your verification solutions and a short discussion that leads the reader through your analysis and results.

Problem 2 (40% of grade)

As shown in Figure 1, an l = 45 inch long hinged gate for an amusement park ride is comprised of a bar of mass m with a concentrated end mass M. The gate rotates about point O and is connected to a torsional spring (k_t) and a torsional damper (b_t), which result in forces proportional to θ and $\dot\theta$, respectively (with θ defined as in Figure 1). The behavior of the system is described by a second order differential equation (note analogy with a linear spring-mass-damper system) as:

$$J_0 \ddot\theta + b_t \dot\theta + k_t \theta = 0$$

where J_0 is the polar moment of inertia of a hinged beam, which for a beam with a concentrated tip mass can be written as
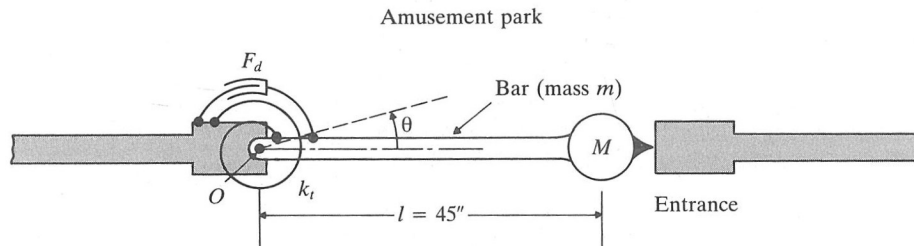
$$J_0 = \frac{1}{3}(m + 4M)l^2$$

Figure 3. Schematic of an amusement park gate.

Part 1. Find values for the torsional spring (k_t) and torsional damper (b_t) with which the gate to closes to within 5° of the equilibrium position (i.e. horizontal) within 2 seconds after being opened to $\theta = 75^o$ (hint: this is one your initial conditions; what is the other?). Assume initially that the concentrated mass is such that $M = 1\ m$.

Part 2. When designing the gate, not only does the designer want it to 'close' quickly, but there is the desire to not make it so stiff that it cannot be opened. In this case, the person entering the gate applied a force F (assumed to be at l = 45") to create a moment on the system.

In this case, the differential equation can now be written in the form

$$J_0 \ddot\theta + b_t \dot\theta + k_t \theta = M(t)$$

where $M(t)$ is the moment applied by the park-goer. (In this case as a first approximation we can assume that the moment is constant as a function of time – ignoring the change in length of the moment arm as the gate swings. If you wish, you can include the change in length of the moment arm in your later calculations as well.)

What is the _force_ that the park-goer needs to apply to open the gate that you designed in Part 1, and is it reasonable? If not, how can you iterate the design to provide an operable gate design?

Report for Problem 2:

You will need to submit an engineering report of your analysis to the company president (i.e. the professor). The report should be **well-organized, clear, and concise**, and at minimum address the points listed below. The report should also provide a justification of why the results that you have obtained are sensible. **NOTE: just submitting the software output without your analysis and discussion is NOT acceptable.**

While it is natural to discuss your work and progress with your colleagues (i.e. classmates), **individual group analyzes and reports are required [this cannot be emphasized enough – work that fails to meet this requirement will not be given credit for the assignment]**. Your supervisor (i.e. the TA) is also available to answer thoughtful questions as you work on the project, but it would be unprofessional to overly rely on your supervisor to complete your project.

- Describe briefly the specifics of the creation of your model in Simulink. In particular mention any aspects that you feel might be 'noteworthy' or unusual.

- Summarize your results. Be sure to compare the results of your model to a simplified 'by hand' calculation(s) to justify your results. BE SURE TO USE RADIANS IN YOUR ANALYSIS.

- Discuss how you found appropriate values for the torsional spring (k_t) and torsional damper (b_t) with which the gate to closes to within 5° in 2 seconds. Show plots of the behavior of the gate as designed.

- Given your solution to Part 1 of the problem, discuss how you determined the force that the park-goer would need to apply to open the gate. Are these values appropriate, or does the gate need to be redesigned?

- How would you optimize the behavior of the gate so that both 'ease of opening' and 'quickness of close' are optimal? Do you think you have such a solution? Why or why not?

- Are their any recommendations to your supervisor regarding 'next steps' in the analysis?

Problem 3 (25% of grade)

FTF Enterprises makes automotive parts, in particular, Camshafts & Gears. The Unit Profit for Camshafts is $25/unit, while the Unit Profit for Gears is $18/unit. To make these products FTF Enterprises needs Steel, Labor, and Machine Time. In total, 5000 lbs of steel are available, 1500 hours of labor are available, and 1000 hours of machine time are available. Camshafts require 5 lbs of steel, 1 hour of labor, and 3 hours machine time, while Gears need 8 lbs of steel per part, 4 hours of labor, and 2 hours machine time.

Part A:

1. Clearly identify your optimization function and list your constraints. What are your design parameters (i.e. what can you change to optimize the problem)?

2. Prepare an analysis in Excel using the optimization/solver tool. (Hint: to check your model, if FTF were to produce 100 camshafts and 100 gears, the total profit would be $4300 with 1300 pounds of steel used.)

3. How many camshafts & gears would you recommend that the company make in order to maximize profit? What is the maximum profit in this case? What parameters are limiting the profits that the company can obtain?

Part B: Due to a shortage of steel, the maximum amount of steel that can be obtained is 2500 pounds. How does this affect the optimal manufacturing numbers?

Part C: A new Gear design has been proposed which will directly replace the existing Gear sold by the company. It is estimated that this Gear will be able to be sold for a Unit Profit of $20/gear. Two manufacturing techniques are being considered: Method 1 will require 4.5 hours of labor and 2 hours of machine time. Method 2 will require 4 hours of labor but 2.25 hours of machine time. Both methods will require 8 pounds of steel per gear. How should the company proceed?

Part D: The analysis above does not include other relevant costs (for example, the cost of the raw materials) into the calculations. By *qualitatively* analyzing the results from the original problem (5000 lbs of steel available, Part A) and the new scenario proposed in Part B (2500 lbs of steel available, Part B), can you make suggestions to the owners of FTF Enterprises on how they may be able to increase their profits? (Hint: how do the profits change for the case when the amount of available steel decreases?)

Report for Problem 3:

In a **SHORT** report for this third part of the Case Study, provide description of your work plus any relevant Excel screen shots, and any conclusions you can make based on your analysis.