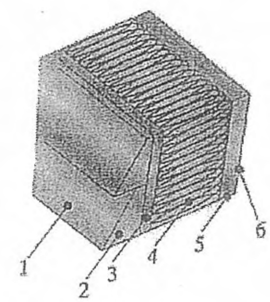
EXAMPLE 1.2

A typical exterior frame wall (made up of 2 × 4 studs) of a house contains the materials shown in the table below. Let us assume an inside room temperature of 70°F and an outside air temperature of 20°F, with an exposed area of 150 ft². We are interested in determining the temperature distribution through the wall.

Items	Resistance hr·ft²·°F/Btu	U-factor Btu/hr·ft²·°F
1. Outside film resistance (winter, 15-mph wind)	0.17	5.88
2. Siding, wood (1/2 × 8 lapped)	0.81	1.23
3. Sheathing (1/2 in regular)	1.32	0.76
4. Insulation batt (3 – 31/2 in)	11.0	0.091
5. Gypsum wall board (1/2 in)	0.45	2.22
6. Inside film resistance (winter)	0.68	1.47



Preprocessing Phase

1. *Discretize the solution domain into finite elements.*
   We will represent this problem by a model that has seven nodes and six elements, as shown in Figure 1.7.
2. *Assume a solution that approximates the behavior of an element.*
   For Example 1.2, there are two modes of heat transfer (conduction and convection) that we must first understand before we can proceed with formulating the conductance matrix and the thermal load matrix. The steady-state thermal behavior of the elements (2), (3), (4), and (5) may be modeled using Fourier's law. When

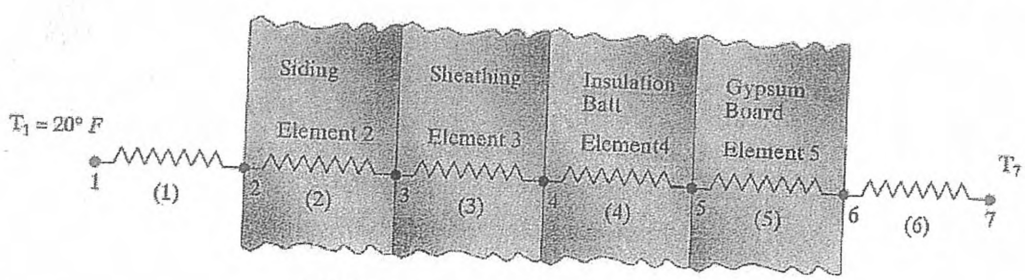


FIGURE 1.7 Finite element model of Example 1.2.

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ME345_FEM_heat_transf1_short

ME345 - Fall2006
Frank's MathCad code to solve equations for heat transfer problem

NOTE changed the 'default' index variable to ONE - so do NOT have subscripts of zero

$$u := \begin{bmatrix} 5.88 \\ 1.23 \\ 0.76 \\ 0.091 \\ 2.22 \\ 1.47 \end{bmatrix}$$

$$bigk := 150 \cdot \begin{bmatrix} u_1 + u_2 & -u_2 & 0 & 0 & 0 \\ -u_2 & u_2 + u_3 & -u_3 & 0 & 0 \\ 0 & -u_3 & u_3 + u_4 & -u_4 & 0 \\ 0 & 0 & -u_4 & u_4 + u_5 & -u_5 \\ 0 & 0 & 0 & -u_5 & u_5 + u_6 \end{bmatrix}$$

$$left\_hand\_side := \begin{bmatrix} 0 + 150 \cdot u_1 \cdot 20 \\ 0 \\ 0 \\ 0 \\ 0 + 150 \cdot u_6 \cdot 70 \end{bmatrix}$$

$$bigk^{-1} \cdot left\_hand\_side = \begin{bmatrix} 20.59 \\ 23.409 \\ 27.972 \\ 66.079 \\ 67.641 \end{bmatrix}$$

ME345_FEM_heat_transf2

ME345 - Fall 2006
Frank's MathCad code to solve equations for heat transfer problem

NOTE changed the 'default' index variable to ONE - so do NOT have subscripts of zero

$$u := \begin{bmatrix} 5.88 \\ 1.23 \\ 0.76 \\ 0.091 \\ 2.22 \\ 1.47 \end{bmatrix}$$

To solve in Matlab, I need to supply initial 'guesses for the solution'

$T2 := 20 \quad T3 := 20 \quad T4 := 20 \quad T5 := 20 \quad T6 := 20$

$q1 := 10 \qquad q7 := 10$

Given

$$\frac{q1}{150} = u_1 \cdot 20 - u_1 \cdot T2$$

$$\frac{0}{150} = -u_1 \cdot 20 + (u_1 + u_2) \cdot T2 - u_2 \cdot T3$$

$$\frac{0}{150} = -u_2 \cdot T2 + (u_2 + u_3) \cdot T3 - u_3 \cdot T4$$

$$\frac{0}{150} = -u_3 \cdot T3 + (u_3 + u_4) \cdot T4 - u_4 \cdot T5$$

$$\frac{0}{150} = -u_4 \cdot T4 + (u_4 + u_5) \cdot T5 - u_5 \cdot T6$$

$$\frac{0}{150} = -u_5 \cdot T5 + (u_5 + u_6) \cdot T6 - u_6 \cdot 70$$

$$\frac{q7}{150} = -u_6 \cdot T6 + u_6 \cdot 70$$

$$\text{find}(T2, T3, T4, T5, T6, q1, q7) = \begin{bmatrix} 20.59 \\ 23.409 \\ 27.972 \\ 66.079 \\ 67.641 \\ -520.161 \\ 520.161 \end{bmatrix}$$