ME 345 Modeling and Simulation
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Summary of finding the state equations for mechanical systems

Goal: The goal is to identify the *state* of a mechanical system, i.e. the equation (singular) or set of equations necessary to describe the evolution (or change in state) of a mechanical system over time. Once the full set of state equations for a system has been determined, one can determine through an appropriate technique (either analytically using the solution of differential equations or numerically using appropriate integration, such as the Euler method).

Five steps to determine the state equations for mechanical systems:

1. <u>Constitutive laws of the elements.</u> Initially we will consider mechanical systems with only the following elements (and assume linear constitutive behavior for the elements)
   a. *Springs:* $f_s = k\, x_s$, where f_s is the force in the spring, k is the spring constant of the spring, and x_s is the elongation of the spring
   b. *Dampers:* $f_d = b\, v_d$, where f_d is the force in the spring, d is the damping constant of the damper, and v_d is the relative velocity (elongation or compression) of the damper
   c. *Masses:* the third type of element is a mass, whose behavior is governed by Newton's law ($\sum F = m\, a_m$).

2. <u>Geometric continuity.</u> For the system we need to ensure consistent relationships between the absolute velocities and the relative velocities of the elements of the system (springs and dampers). The relative velocity of springs and dampers is by definition the difference between the absolute velocities of the "ends" of the elements.

3. <u>Free body diagrams (FBDs).</u> Using Newton's Laws to establish the relationship between the forces acting on different elements in the system.

4. Identify the state variables. The state variables are those variables of the system which need to be prescribed in order to know the current state of the system. For example, consider two *identical* mechanical systems with the same springs, masses, and dampers in the same arrangement; thus the equations of motion of each system is the same. And assume that you take one of the systems and arbitrarily perturb the system in order to set it into motion. In order for me to perturb my system in the exact same manner (I am trying to "synch" my system to exactly match the behavior of yours), I need to know the values of the state variables at a given moment in time. If I can then arrange my system to match those values at the same time, the two systems will be in synch.

   The general rule of thumb is that the MAXIMUM number of state variables in a given system is equal to the number of springs plus the number of masses in the system, and the displacement of the springs (x_s) and the velocity of the masses (v_m) are the state variables. This is the MAXIMUM number because it could be possible that each of these state variables is not independent. For example, if I have two springs in series, if I know the displacement of spring 1 the displacement of spring 2 is also known; in effect, in this case the state equation for each spring will not be independent and the state equation for only one spring is necessary.

5. Solve for the state equations. Here we will use the relationships described in Steps 1-3 to find the state equations for the state variables found in Step 4. The general form is that we want to determine the derivative of the state variable in terms of system element parameters, the state variables (and lower order derivatives of the state variables), and externally applied velocities or forces acting on the system.