# EXAMPLE 3 - Analytical solution

Recall the solution for problem 3 was

$$v_m' = \frac{1}{m}(mg - bv_m) \text{, or...}$$

$$\boxed{v_m' = g - \frac{b}{m} v_m}$$

## Method of Undetermined Coefficients

Assume $\quad v_m = Ae^{rt} + B \quad$ (note: add a constant term because of constant term in DEQ)

Then $\quad v_m' = rAe^{rt} \quad$ (note: derivative of constant is zero!)

If this is a solution, then must satisfy DEQ:

$$rAe^{rt} = g - \left(\frac{b}{m}\right)\left[Ae^{rt} + B\right]$$

Can rewrite:

$$rAe^{rt} = g - \frac{b}{m}Ae^{rt} - \frac{b}{m}B \qquad (*)$$

Let's think about this. The <u>only</u> way this can be true is if the constant and time-dependent terms on the left are identical to those on the right. So...

constants: $0 = g - \frac{b}{m} B$

time-dependent: $rAe^{rt} = -\frac{b}{m} Ae^{rt}$.

Thus we have...

$$0 = g - \frac{b}{m}B \Rightarrow \boxed{B = \frac{mg}{b}}$$

$$rAe^{rt} = -\frac{b}{m}Ae^{rt} \Rightarrow \boxed{r = -b/m}$$

Plug these back into our "guessed" solution...

$$\boxed{V_m = Ae^{-\frac{b}{m}t} + \frac{mg}{b}}$$

General solution!

For a particular solution, we need an initial condition. Here it makes sense that at $t=0$, $V_m = 0$. Thus...

$$0 = A\cancel{e^{-\frac{b}{t}(0)}} + \frac{mg}{b} \Rightarrow \boxed{A = -\frac{mg}{b}.}$$

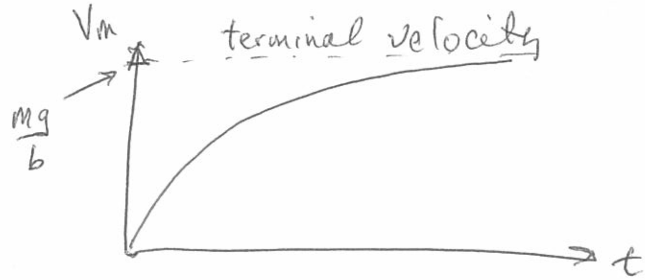
thus $V_M(t) = -\frac{mg}{b} e^{-\frac{b}{m}t} + \frac{mg}{b}$

Can re-write...

$$v_m(t) = \frac{mg}{b}\left(1 - e^{-b/m\, t}\right)$$

← Particular solution for $v_m(t=0) = 0$

Sketch of plot...




Does this plot make sense?

1. At $t = 0$, $v_m = 0$ ✓

2. As time increases, velocity increases towards terminal velocity ✓

3. At terminal velocity, the drag force (damper) is equal to the gravitational force, so $\underline{\underline{\text{no}}}$ acceleration. If no acceleration, velocity becomes constant ✓