Modeling and Simulation for Failure Analysis

Failure Theories

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ME 345

(adapted from materials prepared by Dr. Kishore Pochiraju)

Performance Concerns

- Does it fail under operating conditions?
  - Define Failure mode
    - Static failure
    - Fatigue failure
    - Wear, Ageing or other durability issues

- Static Failure
  - Yield or Break (Maximum Stress < Yield or Failure stress)
  - Deform beyond limit.
    - (Maximum deflection < Specified $\delta_{max}$)
Failure Theories

**MAXIMUM STRESS < MATERIAL STRENGTH**

Problems: Failure theories compare two scalar quantities

**Stress is a tensor, has six components! Failure also needs to be coordinate independent!**

Most Failure Theories make assumptions...

Note: most failure theories are *phenomenological* - not based on atomic level detail

The popular ones are ...

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**Failure of Ductile Materials**

1. The *maximum normal stress* theory - BRITTLE MATERIALS ONLY

2. The *maximum shear stress* theory (also called the Tresca theory) - Good for ductile materials; satisfactory results, easy to use

3. The *maximum strain energy* theory - similar to Von Mises criterion below; better results than above, need Poisson ratio

4. *Von Mises* theory (also called Von Mises - Hencky theory, shear energy theory, and the maximum distortion energy theory) - BEST FOR DUCTILE MATERIALS

From Spotts, Shoup, and Hornberger, *Design of Machine Elements (ME 361)*
1. Maximum Normal (or principle) Stress

- Assumption - failure occurs when any principle stress exceeds the ultimate yield/failure stress from a simple 1D (tensile or compressive) test
- If \( S_{yc} \) is the yield strength in compression, \( S_{yt} \) yield strength in tension

\[
S_{yc} \leq S_1, S_2, S_3 \leq S_{yt}
\]

- To account for safety factor

\[
\frac{S_{yc}}{N_{fs}} \leq S_1, S_2, S_3 \leq \frac{S_{yt}}{N_{fs}}
\]
2. Maximum Shear Stress theory (Tresca)

- Assumption - failure occurs when any principle shear stress exceeds the ultimate yield/failure stress from a simple 1D (tensile or compressive) test
- If $S_y$ is the yield strength

\[-S_y \leq (S_1 - S_2), (S_2 - S_3), (S_1 - S_3) \leq S_y\]

- To account for safety factor

\[-\frac{S_y}{N_{fs}} \leq (S_1 - S_2), (S_2 - S_3), (S_1 - S_3) \leq \frac{S_y}{N_{fs}}\]

3. Maximum strain energy theory

- Failure occurs when strain energy per unit volume exceeds that for a simple uniaxial test at failure
- Derivation in ME358 text - see page 130
- Usually not used as Von Mises failure is simpler and typically more accurate

\[
\frac{S_y}{N_{fs}} = \sqrt{\sigma_1^2 + \sigma_2^2 + \sigma_3^2 - 2\nu(\sigma_1\sigma_2 + \sigma_2\sigma_3 + \sigma_3\sigma_1)}
\]

4. Von Mises theory

- Distortion energy per unit volume in the part is equal to that of a tensile test specimen at failure
- If $S_y$ is the yield strength

\[
\frac{S_y}{N_{fs}} = \sqrt{\sigma_1^2 + \sigma_2^2 + \sigma_3^2 - (\sigma_1\sigma_2 + \sigma_2\sigma_3 + \sigma_3\sigma_1)}
\]

Von Mises Effective Stress
How they fare!