2-sided

25

**PROBLEM 3.**

In class we have discussed the element stiffnesses for both a spring and a rod element, which are given as:

Spring element stiffness matrix:

$$k\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}\begin{Bmatrix} d_1 \\ d_2 \end{Bmatrix}=\begin{Bmatrix} F_1 \\ F_2 \end{Bmatrix}$$

Rod element stiffness matrix:

$$\frac{AE}{L}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}\begin{Bmatrix} d_1 \\ d_2 \end{Bmatrix}=\begin{Bmatrix} F_1 \\ F_2 \end{Bmatrix}$$

We will now consider the torsional element (also referred to as a torsional spring) shown in Figure 4, which has the element spring constant given below

$$\frac{JG}{L}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}\begin{Bmatrix} \theta_1 \\ \theta_2 \end{Bmatrix}=\begin{Bmatrix} T_1 \\ T_2 \end{Bmatrix}$$

where $J$ is the polar moment of inertia, $G$ is the shear modulus, and $L$ is the element length. Note that the torsional spring stiffness relates *nodal angle of twist* $\theta$ to the *nodal applied torque T.*

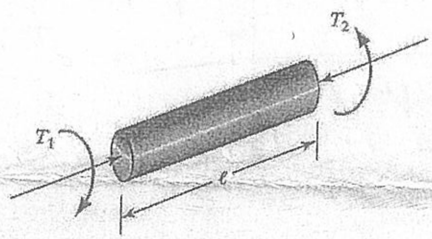



**Figure 4. Torsional spring element.**

**Hint:** recall that for a solid cylindrical rod the polar moment of inertia is given as $J=\frac{1}{2}\pi r^4$ where $r$ is the radius of the rod. *Be careful with units!*

**For the problem shown in For element 1: r1 = 2 inches, L1 = 2 ft, G1 = 3 * $10^6$ lb/in$^2$**

**For element 2: r2 = 1 inch, L2 = 1 ft, G2 = 3 * $10^6$ lb/in$^2$**

Figure 5a (clearly show and label all work):

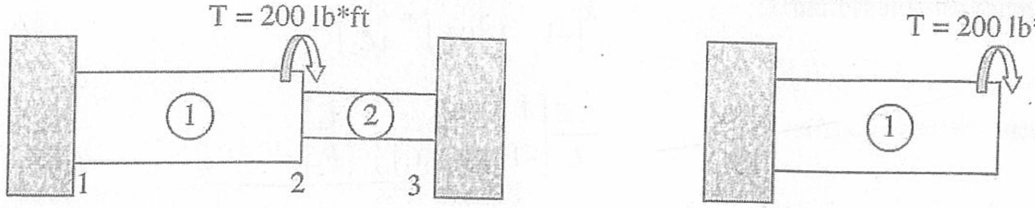
1. assemble the *global* stiffness matrix.
2. solve for the angle of twist at node 2.
3. justify your analysis by considering the angle of twist for a torque of 200 lb*ft applied on only element 1. (i.e. see Figure 5b)

Part 3 → Comparison:

$$\theta = \frac{TL}{JG} = \frac{(200\ lb\ ft)(\frac{12\ in}{1ft})(2ft)(\frac{12\ in}{1ft})}{(8\pi\ in^4)(3\times10^6\ lb/in^2)}$$

$$\theta = 7.64\times10^{-4}\ rad$$

this is more because less resistance to twist in this case!




**For element 1: r1 = 2 inches, L1 = 2 ft, G1 = 3 * 10⁶ lb/in²**

**For element 2: r2 = 1 inch, L2 = 1 ft, G2 = 3 * 10⁶ lb/in²**

**Figure 5a. (left) Torsional problem via the finite element method. (right) Comparison problem.**

Ele 1: $\frac{JG}{L} = \frac{(8\pi\ in^4)(3\times10^6\ lb/in^2)}{2ft\times\frac{12 in}{1ft}} = \pi\times10^6\ lb\cdot in \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$

$J_1 = \frac{1}{2}\pi(2)^4 = 8\pi\ in^4$

Ele 2: $\frac{JG}{L} = \frac{(\frac{\pi}{2}\ in^4)(3\times10^6\ lb/in^2)}{1ft\times\frac{12 in}{1ft}} = \frac{\pi}{8}\times10^6\ lb\cdot in \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$

$J_2 = \frac{1}{2}\pi(1)^4 = \frac{\pi}{2}\ in^4$

GLOBAL STIFFNESS MATRIX

$$\begin{Bmatrix} T_1 \\ T_2 \\ T_3 \end{Bmatrix} = \pi\times10^6\ lb\cdot in \begin{bmatrix} 1 & -1 & 0 \\ -1 & 1+\frac{1}{8} & -\frac{1}{8} \\ 0 & -\frac{1}{8} & \frac{1}{8} \end{bmatrix} \begin{Bmatrix} \theta_1 \\ \theta_2 \\ \theta_3 \end{Bmatrix}$$

$$T_2 = \frac{9}{8}\pi\times10^6\ \theta_2$$

$$200\ lb\cdot ft\times\frac{12 in}{1ft} = (\tfrac{9}{8}\pi\times10^6)(\theta_2) \Rightarrow \theta_2 = 6.79\times10^{-4}\ rad$$

angle of twist at node 2