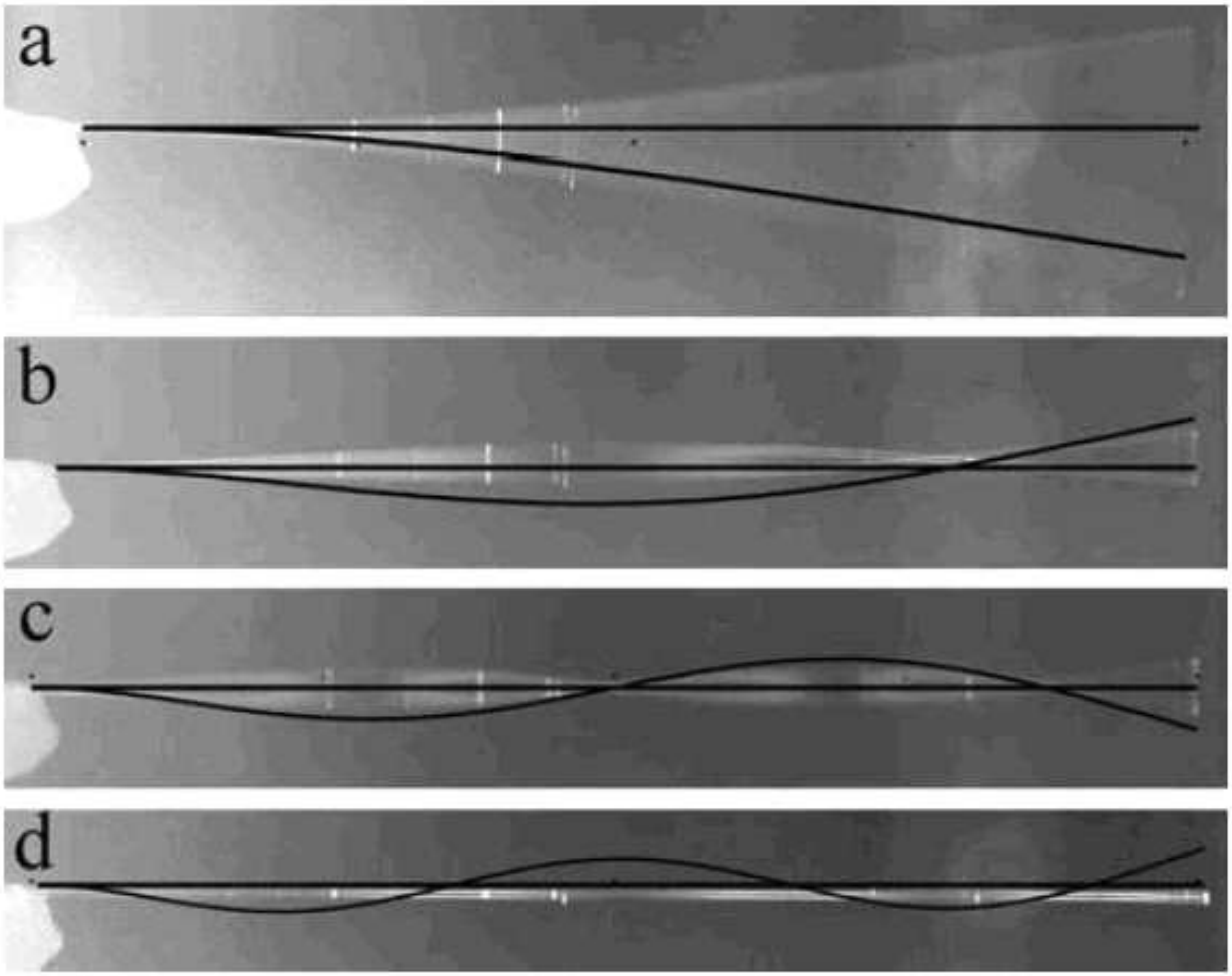
Vibration / Modal Analysis



Shown: experimentally determining the modulus of nanofibers via free vibration behavior

Beam Vibration

TABLE 7.3 Natural Frequencies and Normal Modes of Uniform Beams

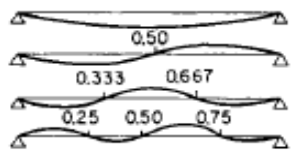
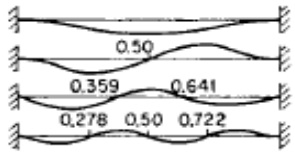
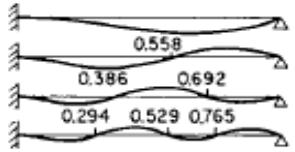
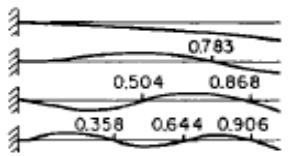
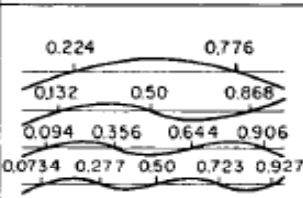
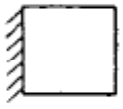
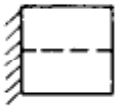
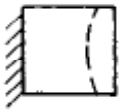
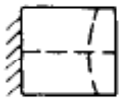
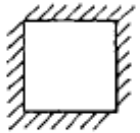
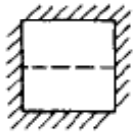
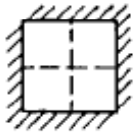
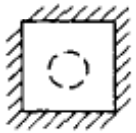
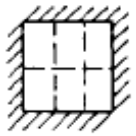
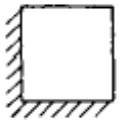
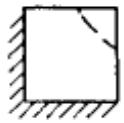
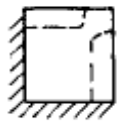
SUPPORTS	MODE n	(A) SHAPE AND NODES (NUMBERS GIVE LOCATION OF NODES IN FRACTION OF LENGTH FROM LEFT END)	(B) BOUNDARY CONDITIONS EQ (7.16)	(C) FREQUENCY EQUATION	(D) CONSTANTS EQ (7.16)	(E) kl EQ (7.14) $\omega_n = k^2\sqrt{\frac{EIg}{A\gamma}}$	(F) R RATIO OF NON-ZERO CONSTANTS COLUMN (D)
HINGED-HINGED	1		$x=0\begin{cases}X=0\\X''=0\end{cases}$ $x=l\begin{cases}X=0\\X''=0\end{cases}$	SIN kl=0	A = 0	3.1416	1.0000
	2				B = 0	6.283	1.0000
	3				$\frac{C}{D}=1$	9.425	1.0000
	4					12.566	1.0000
	n>4					$\approx n\pi$	1.0000
CLAMPED-CLAMPED	1		$x=0\begin{cases}X=0\\X'=0\end{cases}$ $x=l\begin{cases}X=0\\X'=0\end{cases}$	(COS kl) (COSH kl) = 1	A = 0	4.730	-0.9825
	2				C = 0	7.853	-1.0008
	3				$\frac{D}{B}=R$	10.996	-1.0000-
	4					14.137	-1.0000+
	n>4					$\approx\frac{(2n+1)\pi}{2}$	-1.0000-
CLAMPED-HINGED	1		$x=0\begin{cases}X=0\\X'=0\end{cases}$ $x=l\begin{cases}X=0\\X''=0\end{cases}$	TAN kl = TANH kl	A = 0	3.927	-1.0008
	2				C = 0	7.069	-1.0000+
	3				$\frac{D}{B}=R$	10.210	-1.0000
	4					13.352	-1.0000
	n>4					$\approx\frac{(4n+1)\pi}{4}$	-1.0000
CLAMPED-FREE	1		$x=0\begin{cases}X=0\\X'=0\end{cases}$ $x=l\begin{cases}X''=0\\X'''=0\end{cases}$	(COS kl) (COSH kl) = -1	A = 0	1.875	-0.7341
	2				C = 0	4.694	-1.0185
	3				$\frac{D}{B}=R$	7.855	-0.9992
	4					10.996	-1.0000+
	n>4					$\approx\frac{(2n-1)\pi}{2}$	-1.0000-
FREE-FREE	1		$x=0\begin{cases}X''=0\\X'''=0\end{cases}$ $x=l\begin{cases}X''=0\\X'''=0\end{cases}$	(COS kl) (COSH kl) = 1	B = 0	0 (REPRESENTS TRANSLATION)	
	2				D = 0	4.730	-0.9825
	3				$\frac{C}{A}=R$	7.853	-1.0008
	4					10.996	-1.0000-
	5					14.137	-1.0000+
	n>5					$\approx\frac{(2n-1)\pi}{2}$	-1.0000-

Plate Vibration

TABLE 7.7 Natural Frequencies and Nodal Lines of Square Plates with Various Edge Conditions (*After D. Young.*[29])

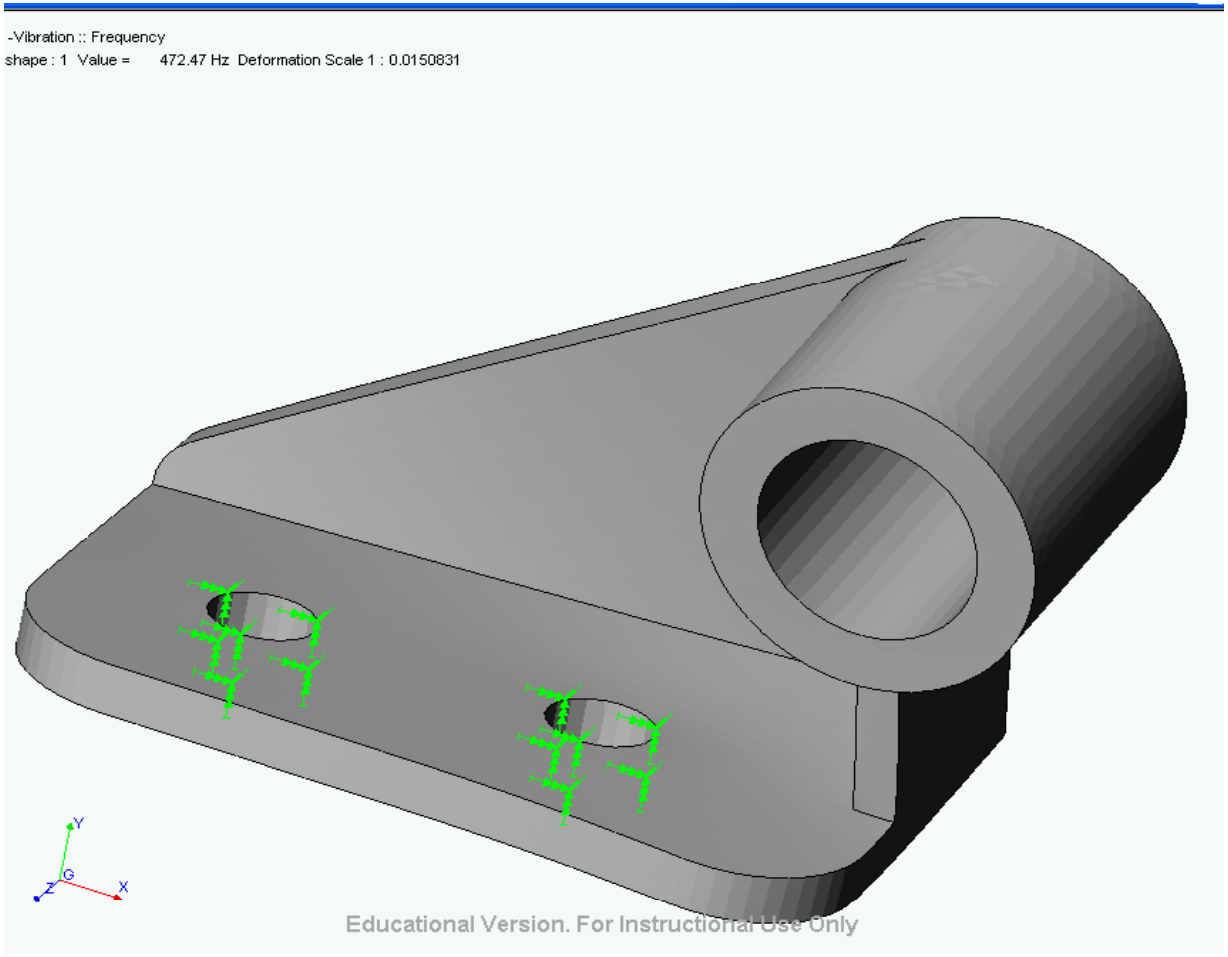
	1ST MODE	2ND MODE	3RD MODE	4TH MODE	5TH MODE	6TH MODE
$\omega_n/\sqrt{Dg/\gamma h a^4}$	3.494	8.547	21.44	27.46	31.17	
NODAL LINES						
$\omega_n/\sqrt{Dg/\gamma h a^4}$	35.99	73.41	108.27	131.64	132.25	165.15
NODAL LINES						
$\omega_n/\sqrt{Dg/\gamma h a^4}$	6.958	24.08	26.80	48.05	63.14	
NODAL LINES						

$\omega_n = 2\pi f_n$ h = PLATE THICKNESS

$D = Eh^3/12(1-\mu^2)$ a = PLATE LENGTH

γ = WEIGHT DENSITY

More Complex Geometry

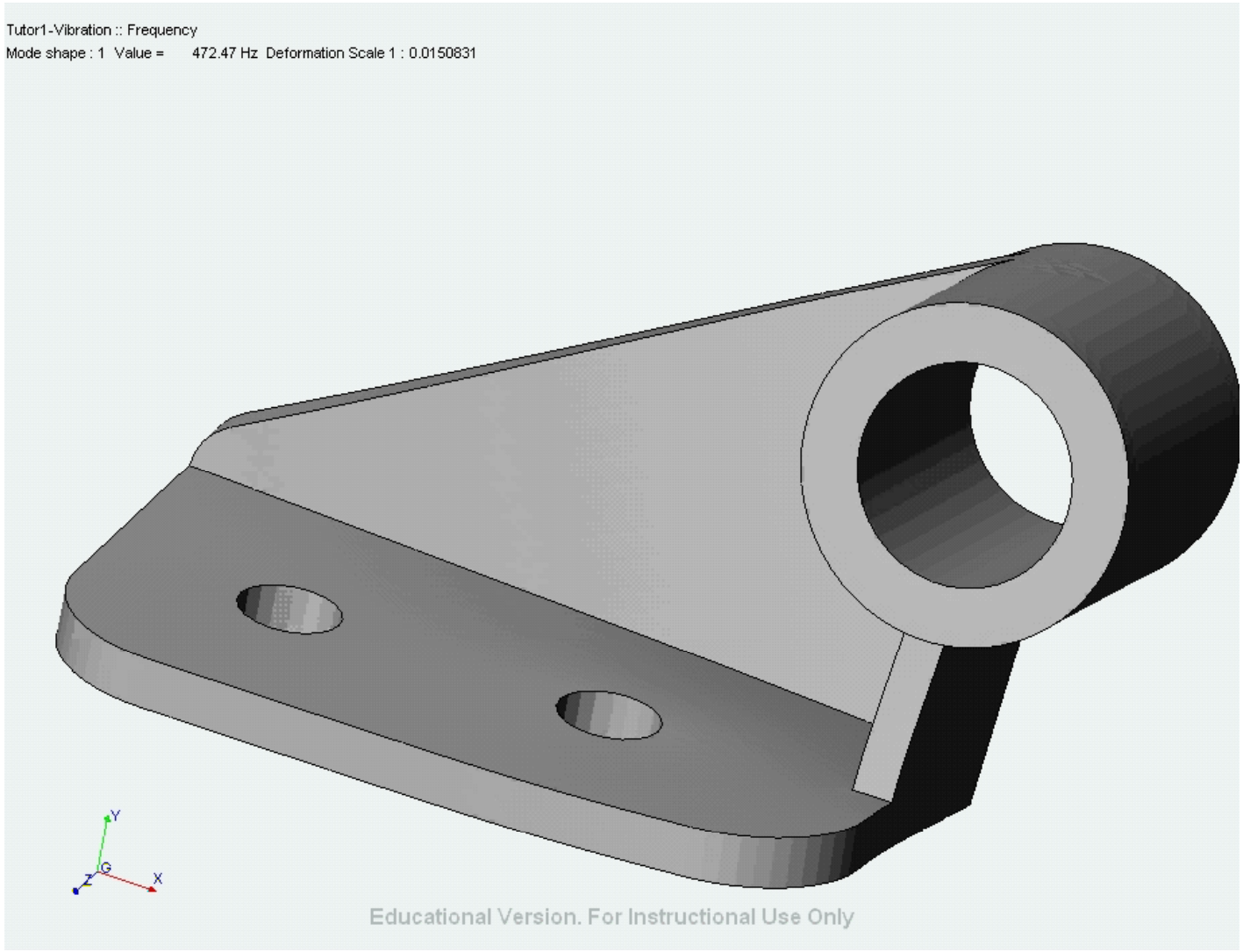


Example Problem

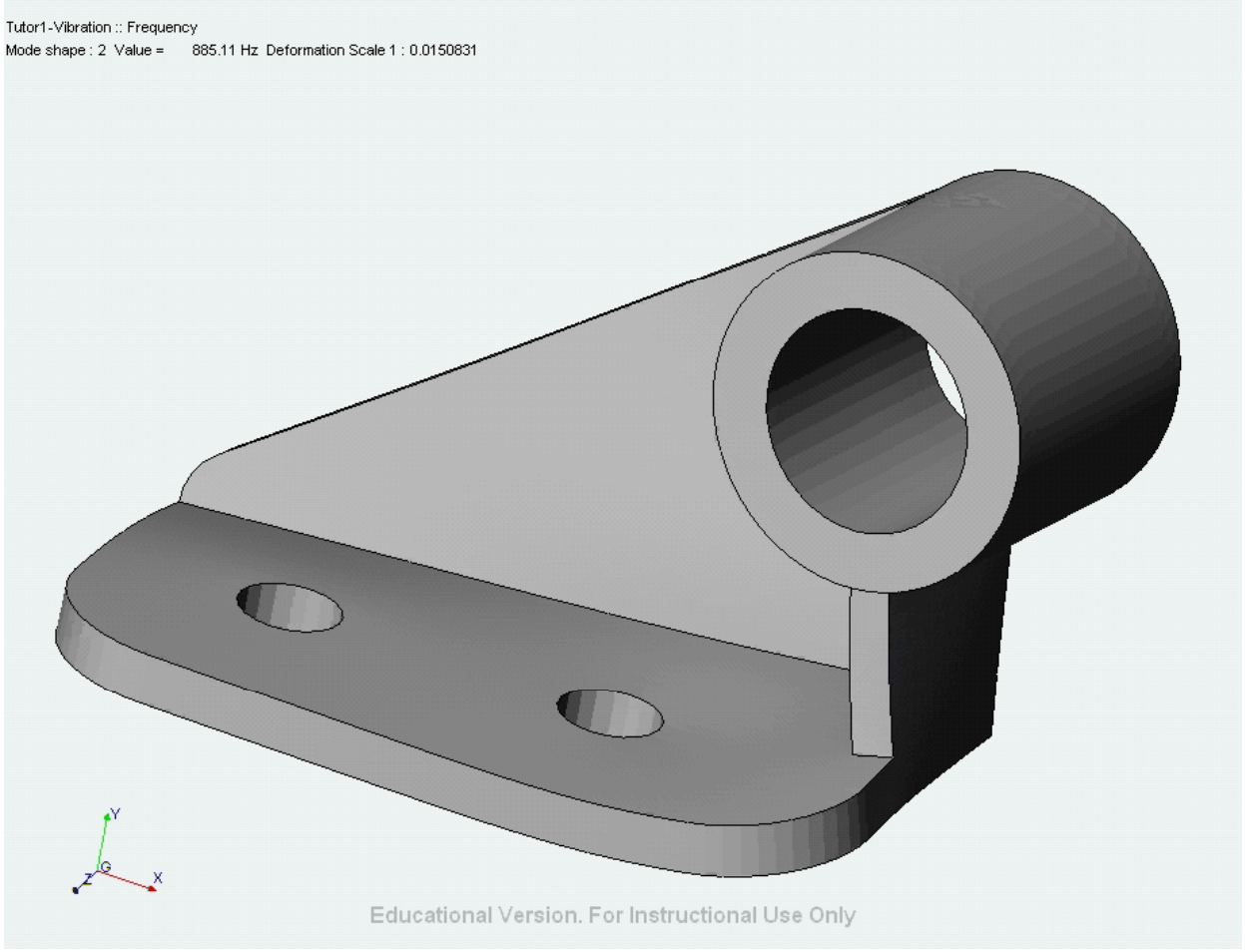
- Natural Frequencies

Frequency Number	Hertz	Seconds
1	472.47	0.0021165
2	885.11	0.0011298
3	1492.1	0.00067018
4	2238.4	0.00044675
5	3989.3	0.00025067
6	4407.7	0.00022688
7	5167.5	0.00019352
8	5638	0.00017737
9	6834.9	0.00014631
10	6966.8	0.00014354
11	7391.3	0.00013529
12	7894.8	0.00012667
13	9065.1	0.00011031
14	9807.1	0.00010197
15	10363	9.6498e-005

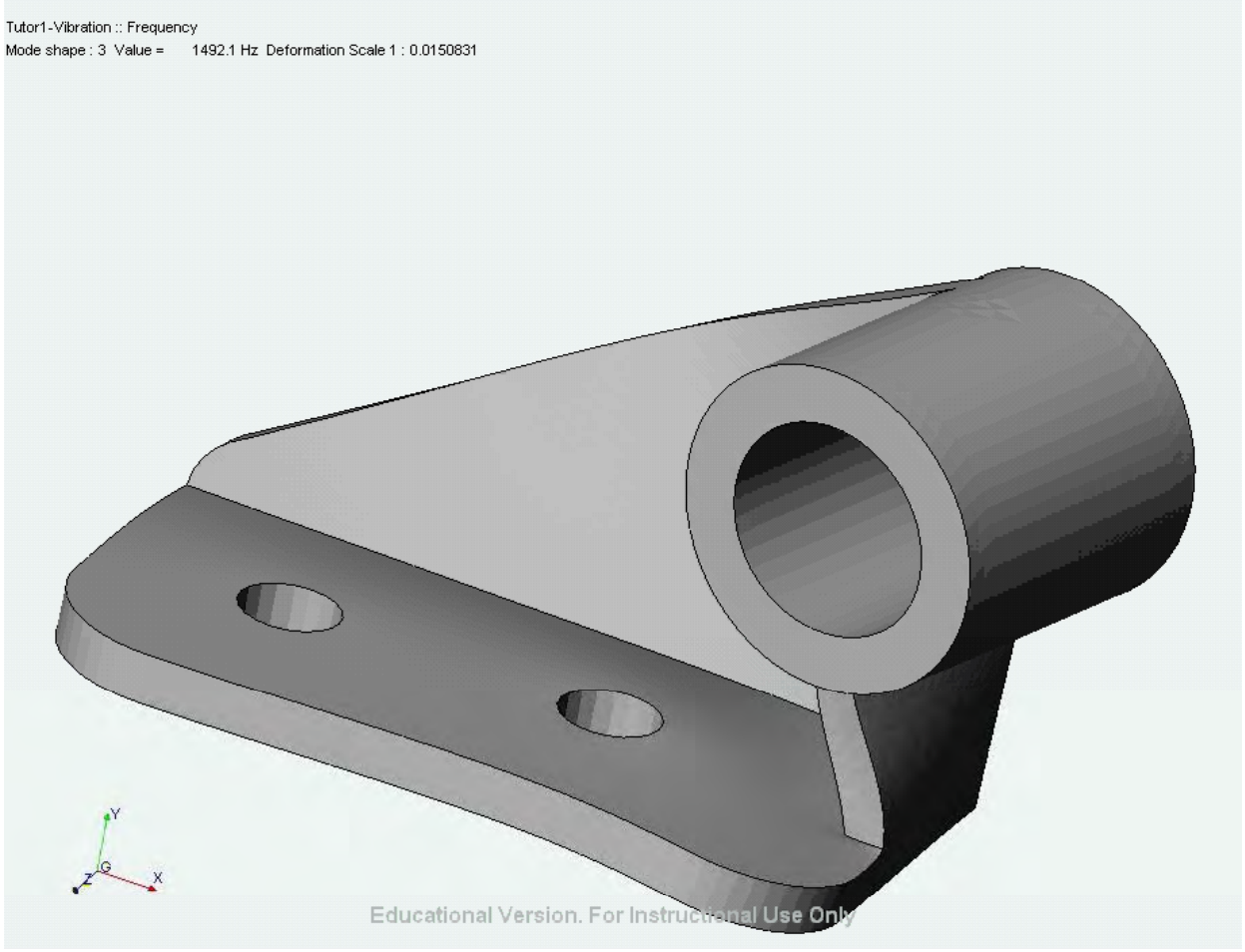
Mode - I


Tutor1-Vibration :: Frequency
Mode shape : 1 Value = 472.47 Hz Deformation Scale 1 : 0.0150831
Y
G
Z
X
Educational Version. For Instructional Use Only

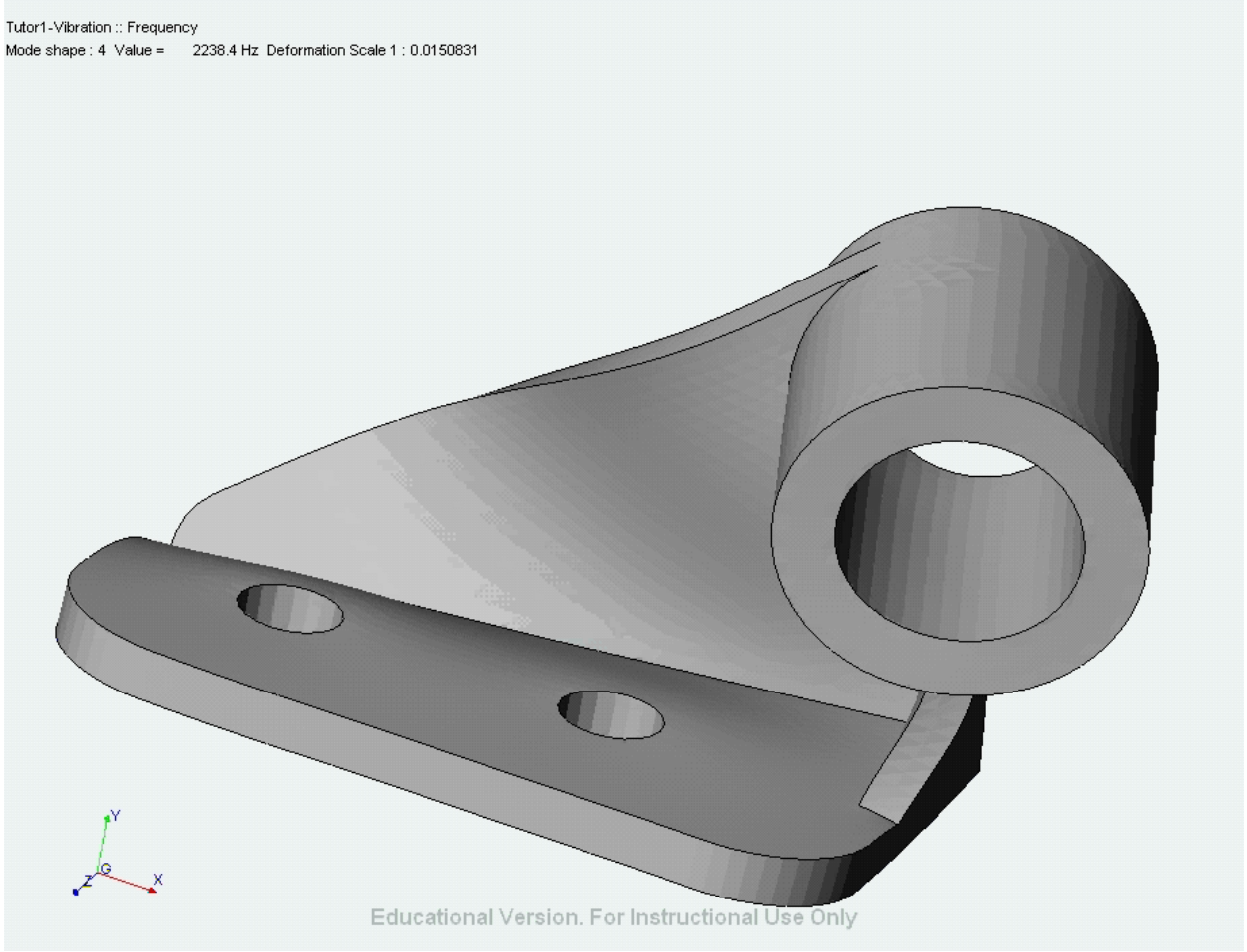
Mode - II


Tutor1-Vibration :: Frequency
Mode shape : 2 Value = 885.11 Hz Deformation Scale 1 : 0.0150831
Y
G
Z
X
Educational Version. For Instructional Use Only

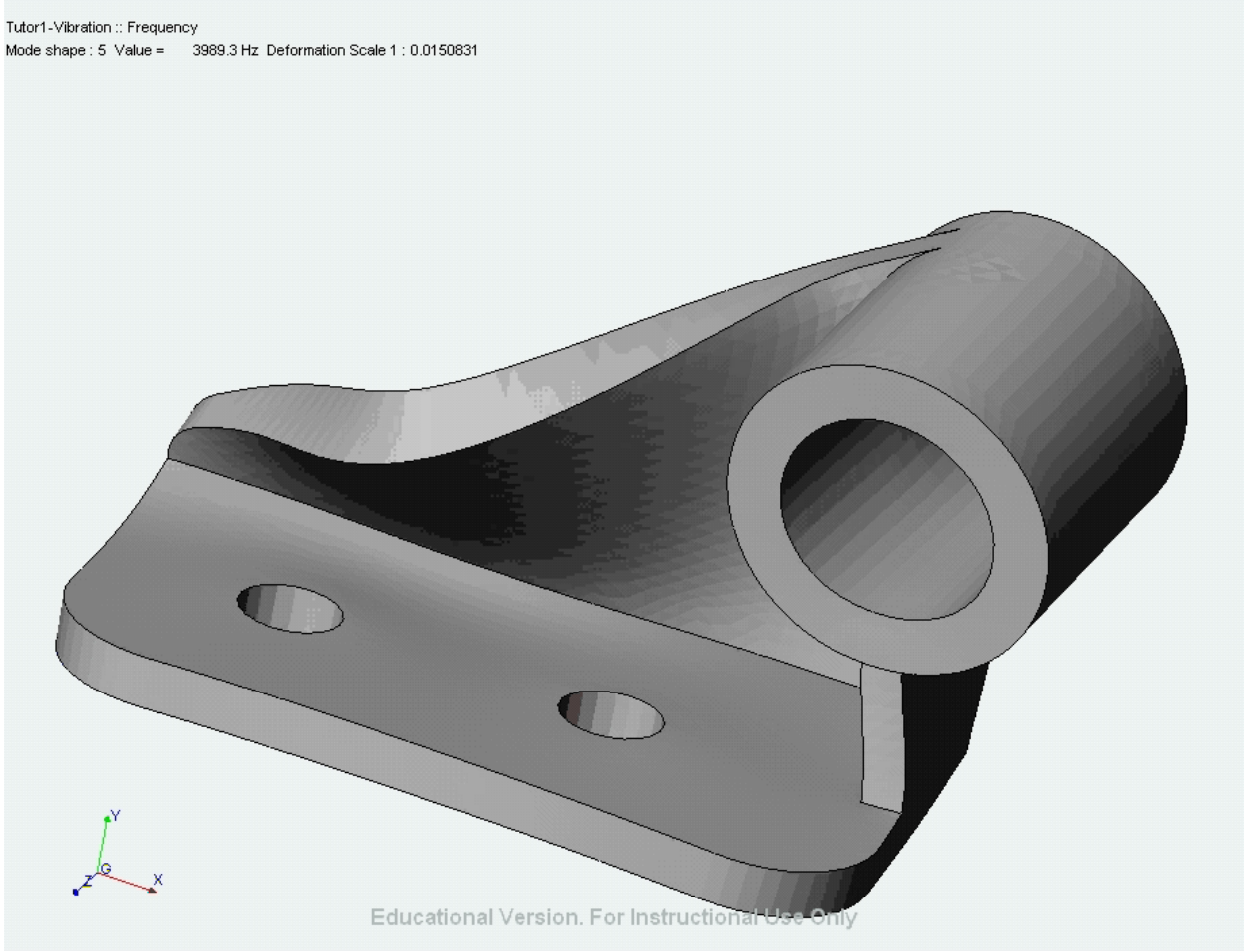
Mode - III


Tutor1-Vibration :: Frequency
Mode shape : 3 Value = 1492.1 Hz Deformation Scale 1 : 0.0150831
Y
G
Z
X
Educational Version. For Instructional Use Only

Mode - IV


Tutor1-Vibration :: Frequency
Mode shape : 4 Value = 2238.4 Hz Deformation Scale 1 : 0.0150831
Y
G
Z
X
Educational Version. For Instructional Use Only

Mode - V


Tutor1-Vibration :: Frequency
Mode shape : 5 Value = 3989.3 Hz Deformation Scale 1 : 0.0150831
Y
G
Z
X
Educational Version. For Instructional Use Only