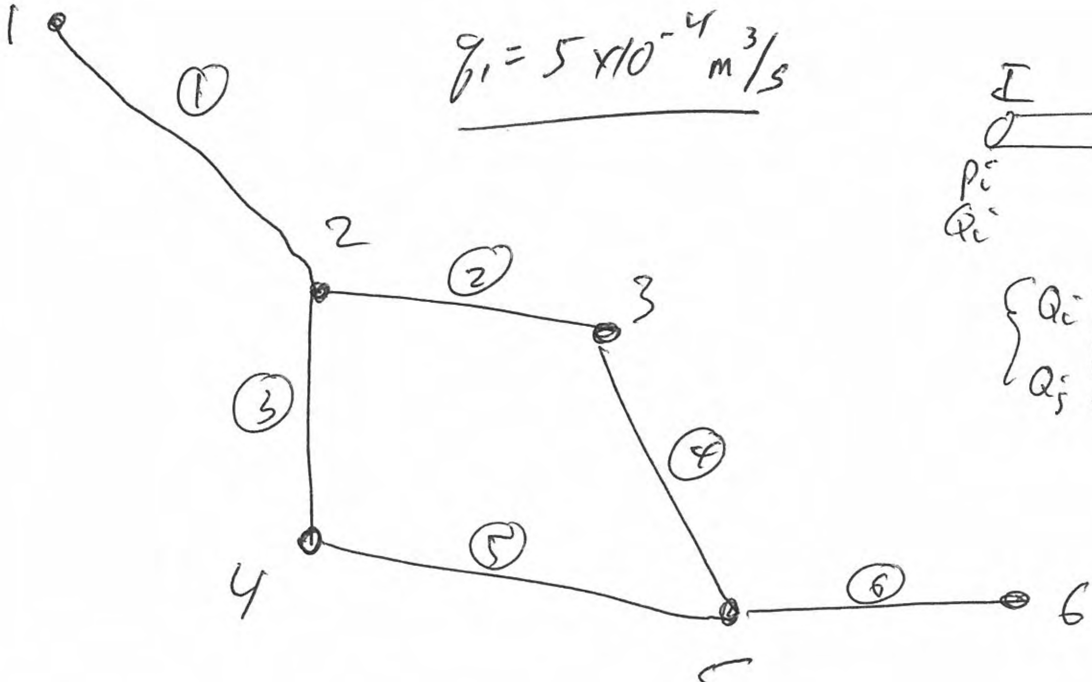
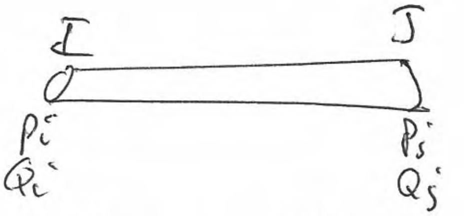


$$q_1 = 5 \times 10^{-4}\ m^3/s$$

①



$$\begin{Bmatrix} Q_i^i \\ Q_j^i \end{Bmatrix} = \frac{\pi D^4}{128 L \mu} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{Bmatrix} p_i^i \\ p_j^i \end{Bmatrix}$$

$$\mu = 0.3\ Ns/m^2$$

$$\frac{[m^4]}{[m]\left[\frac{Ns}{m^2}\right]} = \frac{m^5}{N.s}$$

Ele #1: $\dfrac{(\pi)(0.1)^4}{((128)(70.71)(0.3))} = 1.157 \times 10^{-7}\ \dfrac{m^5}{N \cdot s.}$ $= R_1$

note: $\left[\dfrac{m^5}{N \cdot s}\right]\left[\dfrac{N}{m^2}\right] = \dfrac{m^3}{s}$

↑ units of pressure

↑ units of volumetric flow

✓ units check

ELE #2 $\dfrac{(\pi)(.075)^4}{(128)(50.99)(0.3)} = 0.5077 \times 10^{-7}\ m^5/N.S.$ $= R_2$

Ele #3 $\dfrac{(\pi)(0.75)^4}{(128)(50)(0.3)} = 0.5177 \times 10^{-7}\ m^5/N.S$ $= R_3$

Ele #4

$$\frac{(\pi)(0.05)^4}{(128)(53.85)(0.3)} = .095 \times 10^{-7} \ m^5/NS = R_4$$

Ele #5

$$\frac{(\pi)(.05)^4}{(128)(70.71)(0.3)} = 0.0723 \times 10^{-7} \ m^5/N.s = R_5$$

Ele #6

$$\frac{(\pi)(.1)^4}{(128)(60)(.3)} = 1.3635 \times 10^{-7} \ m^5/N.s = R_6$$

NOW ASSEMBLE GLOBAL STIFFNESS MATRIX. NOTE THAT THERE ARE 6 nodes and 6 elements.

$$\begin{bmatrix} Q_1 \\ Q_2 \\ Q_3 \\ Q_4 \\ Q_5 \\ \cancel{Q_6} \end{bmatrix} = \begin{bmatrix} R_1 & -R_1 & 0 & 0 & 0 & 0 \\ -R_1 & R_1+R_2+R_3 & -R_2 & -R_3 & 0 & 0 \\ 0 & -R_2 & R_2+R_4 & 0 & -R_4 & 0 \\ 0 & -R_3 & 0 & R_3+R_5 & -R_5 & 0 \\ 0 & 0 & -R_4 & -R_5 & R_4+R_5+R_6 & -R_6 \\ \cancel{0} & \cancel{0} & \cancel{0} & \cancel{0} & \cancel{-R_6} & \cancel{R_6} \end{bmatrix} \begin{bmatrix} P_1 \\ P_2 \\ P_3 \\ P_4 \\ P_5 \\ \cancel{P_6} = 0 \end{bmatrix}$$

GLOBAL STIFFNESS MATRIX

③

Now apply the Boundary Conditions. Note that Q_i are the "external flow sources", so $Q_1 = 5\times10^{-4}\ m^3/s$ and Q_2 to Q_5 are zero. (Q_6 should equal $-Q_1$ since steady state, which will be a check at the end.)

$$\begin{bmatrix} \cancel{Q_1}^{5\times10^{-4}} \\ \cancel{Q_2}^{0} \\ \cancel{Q_3}^{0} \\ \cancel{Q_4}^{0} \\ \cancel{Q_5}^{0} \end{bmatrix} = 10^{-9} \begin{bmatrix} 115.7 & -115.7 & 0 & 0 & 0 \\ -115.7 & 28.23 & -50.76 & -51.77 & 0 \\ 0 & -50.76 & 60.26 & 0 & -9.5 \\ 0 & -51.77 & 0 & 59.0 & -7.23 \\ 0 & 0 & -9.5 & -7.23 & 153.08 \end{bmatrix} \begin{bmatrix} P_1 \\ P_2 \\ P_3 \\ P_4 \\ P_5 \end{bmatrix}$$

Solve via Matlab (see attached code).

$$\begin{bmatrix} P_1 \\ P_2 \\ P_3 \\ P_4 \\ P_5 \end{bmatrix} = \begin{bmatrix} 4.2841 \\ 3.8519 \\ 3.3025 \\ 3.4248 \\ 0.3667 \end{bmatrix} \times 10^4\ N/m^2$$

To checks, lets take those five pressures (p_1, p_2, p_3, p_4, and p_5) and the fact that we set $p_6 = 0$ and apply back into our 6x6 GLOBAL STIFFNESS MATRIX. If multiply out, should get our known (steady state) flows.

See attached Matlab file.

$$[Q_1, Q_2, Q_3, Q_4, Q_5, Q_6]' = [5.0, 0, 0, 0, 0, -5]' \times 10^{-4} \ m^3/s$$

Checks!

Can now calculate the flows in each pipe using the relationship

$$q_i = \frac{\pi D^4}{128 L \mu} (p_i - p_j)$$

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$$q_1 = \frac{(\pi)(0.1)^4}{(128)(40.71)(0.3)}(4.2841 - 3.8579)\times 10^4 = 5\times 10^{-4}\ m^3/s$$

(note: another check!)

$$q_2 = \frac{(\pi)(.075)^4}{(128)(50.99)(0.3)} = 2.789\times 10^{-4}\ m^3/s$$

$$q_3 = \frac{(\pi)(.075)^4}{(128)(50)(0.3)} = 2.211\times 10^{-4}\ m^3/s$$

Using similar relationships...

$q_4 = 2.789\times 10^{-4}\ m^3/s$

$q_5 = 2.211\times 10^{-4}\ m^3/s$

$q_6 = 5.00\times 10^{-4}\ m^3/s$ ← this is the flow in Element 6 that is leaving the system. Needs to be equal to Q_1 since steady-state!

NOTE: $\left.\begin{array}{l} q_2 = q_4 \\ q_3 = q_5 \end{array}\right\}$ makes sense; see geometry