Internal Pipeflow Problem Using a Finite Element Approach

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ME345: Modeling and Simulation

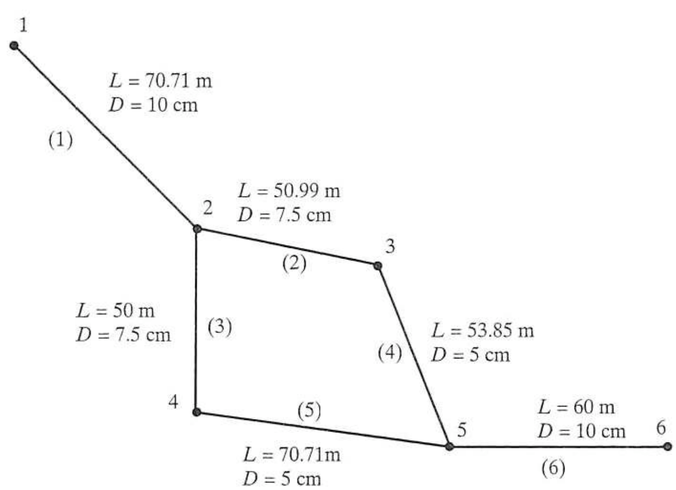
For internal pipe flow, one can derive a finite element formulation that relates the *volumetric flow rate at a point Q [units of m^3/s]* to the *nodal pressures p [units of Pa]* via an element stiffness matrix of the form

$$\frac{\pi D^4}{128 L \mu}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}\begin{Bmatrix} p_1 \\ p_2 \end{Bmatrix} = \begin{Bmatrix} Q_1 \\ Q_2 \end{Bmatrix}$$

where D is the internal diameter of the pipe, L is the length of the element, and μ is the dynamic viscosity of the fluid [units of s/m^2]. (The above relationship holds true for simple, laminar flows.) The coefficient $(\pi D^4)/(128 L \mu)$ is sometimes known as the *flow resistance* [units of m^5 / N s.

Using this definition of 1D pipe flow, analyze the following piping schematic below given the following known (boundary) conditions:

$Q_1 = Q_{inlet} = 5 \times 10^{-4}\ m^3/s$
$\mu = 0.3$ N s / m^2
$p_6 = 0$ Pa[1]



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[1] *Here we are treating p_6 as at atmospheric pressure, and can assign it an arbitrary value; setting it equal to zero is the easiest value to set it to here. All other pressures determined will be relative to this reference pressure.*