Internal Pipeflow Problem Using a Finite Element Approach

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For internal pipe flow, one can derive a finite element formulation that relates the volumetric flow rate at a point $Q$ [units of $m^3/s$] to the nodal pressures $p$ [units of Pa] via an element stiffness matrix of the form

$$\frac{\pi D^4}{128 L \mu} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} p_1 \\ p_2 \end{bmatrix} = \begin{bmatrix} Q_1 \\ Q_2 \end{bmatrix}$$

where $D$ is the internal diameter of the pipe, $L$ is the length of the element, and $\mu$ is the dynamic viscosity of the fluid [units of $s/m^2$]. (The above relationship holds true for simple, laminar flows.) The coefficient $\left(\frac{\pi D^4}{128 L \mu}\right)$ is sometimes known as the flow resistance [units of $m^5 / N \cdot s$].

Using this definition of 1D pipe flow, analyze the following piping schematic below given the following known (boundary) conditions:

$Q_1 = Q_{\text{inlet}} = 5 \times 10^{-4} \text{ m}^3/\text{s}$
$\mu = 0.3 \text{ N} \cdot \text{s} / \text{m}^2$
$p_0 = 0 \text{ Pa}$

\[\text{Diagram of piping system}\]

\[1\] Here we are treating $p_0$ as at atmospheric pressure, and can assign it an arbitrary value; setting it equal to zero is the easiest value to set it to here. All other pressures determined will be relative to this reference pressure.