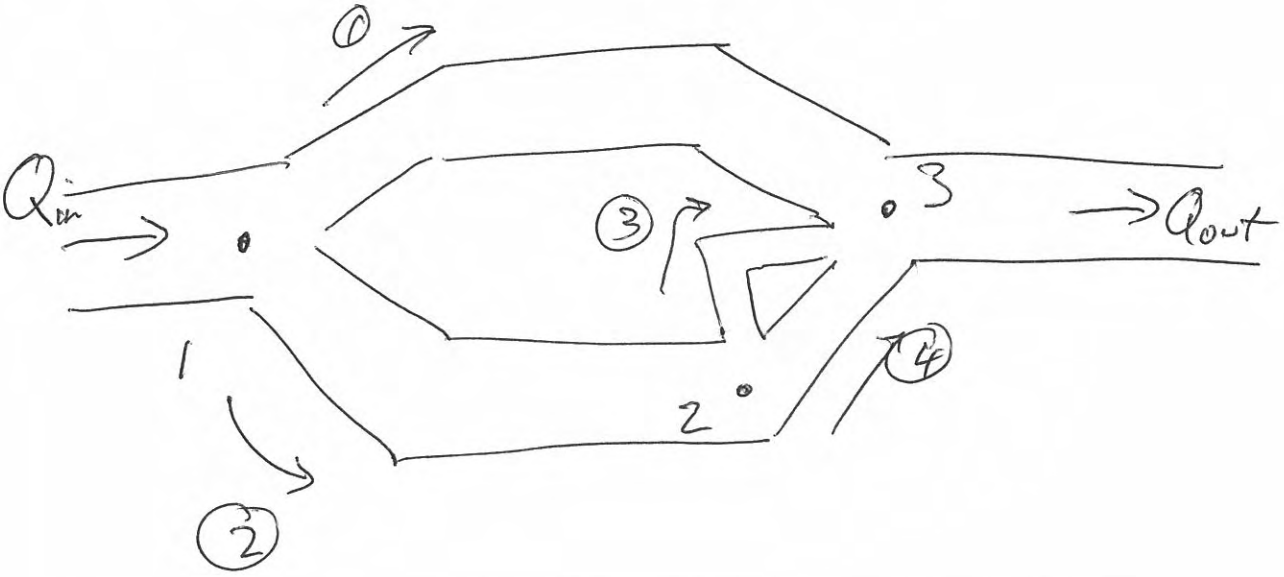
Internal Pipe Flow - Example 2



Note: at steady state, $Q_{in} = Q_{out}$.

$$\begin{Bmatrix} Q_i \\ Q_j \end{Bmatrix} = \frac{\pi D^4}{128 L \mu} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{Bmatrix} P_i \\ P_j \end{Bmatrix}$$

Network details:

$Q_{in} = 0.1\ m^3/s$

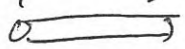
$D_1 = 2.5\ cm$ $L_1 = 30\ m$

$D_2 = 2.0\ cm$ $L_2 = 20\ m$

$D_3 = 2.0\ cm$ $L_3 = 25\ m$

$D_4 = 1.25\ cm$ $L_4 = 20\ m$

Element 1 : $\frac{\pi D^4}{128 L \mu} = \frac{(\pi/(.025)^4}{(128)(30)(0.96\times10^{-3})} = 3.33\times10^{-7}$ m^5/Ns (R_1)



Element 2 : $\frac{\pi D^4}{128 L \mu} = \frac{(\pi/(.02)^4}{(128)(20)(.96\times10^{-3})} = 2.05\times10^{-7}$ m^5/Ns (R_2)

Element 3: $\frac{\pi D^4}{128 L \mu} = \frac{(\pi/(.02)^4}{(128)(25)(.96\times10^{-3})} = 1.64\times10^{-7}$ m^5/Ns (R_3)

Element 4: $\frac{\pi D^4}{128 L \mu} = \frac{(\pi/(.0125)^4}{(128)(20)(.96\times10^{-3})} = 3.12\times10^{-8}$ m^5/Ns (R_4)

To simply notation, use R_1-R_4 to refer to the respective flow resistance in each element.

Now assemble the global stiffness matrix. Note that there are 3 nodes and 4 elements

$$\begin{bmatrix} Q_1 \\ Q_2 \\ Q_3 \end{bmatrix} = \begin{bmatrix} R_1+R_2 & -R_2 & -R_1 \\ -R_2 & R_2+R_3+R_4 & -R_3-R_4 \\ -R_1 & -R_3-R_4 & R_1+R_3+R_4 \end{bmatrix} \begin{bmatrix} P_1 \\ P_2 \\ P_3 \end{bmatrix}$$

GLOBAL STIFFNESS MATRIX

Now apply BC's. $Q_1 = 0.1\ m^3/s$, and assume $P_3 = 0$ (assume relative pressure is zero). This gives us...

$$\begin{bmatrix} 0.1 \\ 0 \end{bmatrix} = \begin{bmatrix} R_1+R_2 & -R_2 \\ -R_2 & R_2+R_3+R_4 \end{bmatrix} \begin{bmatrix} P_1 \\ P_2 \end{bmatrix}$$

$$P_1 = 230952\ Pa$$

$$P_2 = 118303\ Pa$$

($P_3 = 0$ Pa by assumption → relative pressure!)

To get the internal flow in each pipe...

$$Q = \frac{\pi D^4}{(128)(L)(\mu)} (P_i - P_{i+1})$$

$$q_1 = \left(3.33 \times 10^{-7} \frac{m^5}{N \cdot s}\right)\left(230{,}952 \frac{N}{m^2} - 0\right) = .077 \text{ } m^3/s$$

$$q_2 = \left(2.05 \times 10^{-7} \frac{m^5}{N s}\right)(230{,}952 - 118{,}303) = .023 \text{ } m^3/s$$

$$q_3 = \left(1.64 \times 10^{-7} \frac{m^5}{N \cdot s}\right)(118{,}303 - 0) = .019 \text{ } m^3/s$$

$$q_4 = \left(3.12 \times 10^{-8} \frac{m^5}{N \cdot s}\right)(118{,}303 - 0) = .004 \text{ } m^3/s.$$

Checks. $q_1 + q_2 = 0.1 \text{ } m^3/s$ ✓

$q_2 = q_3 + q_4$ ✓

$q_1 + q_3 + q_4 = 0.1 \text{ } m^3/s$ ✓