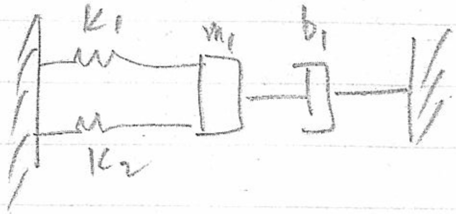
EXAMPLE 6 (spring-mass-damper)

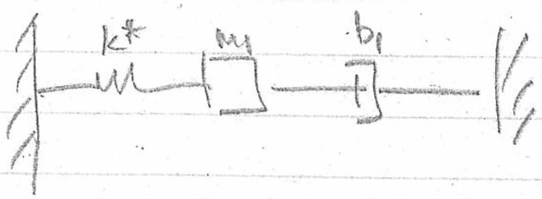
~~EXAMPLE 6~~



Find: state equations

NOTE: ON inspection, you could see that k_1 and k_2 are in parallel, and simplify to the system below. In class, we showed how to work through without doing this.

EQUIVALENT SYSTEM



$\boxed{k^* = k_1 + k_2}$

1. $f_s = k^* x_s \qquad f_b = b_1 v_d$

2. $v_s = v_m \qquad v_b = -v_m$

3. 

$\Sigma F_x = ma_x$

$-f_s + f_b = ma_m$

4. SVs: x_s, v_m (note $x_s = x_{s1} = x_{s2}$)

5. $x_s' = v_s = v_m$

$$v_m' = a_m = \frac{1}{m}(-f_s + f_d) = \frac{1}{m}(-k^* x_s + b v_d)$$

$$= \frac{1}{m}(-k^* x_s - b v_m)$$

Above are two first order state equations. Can combine to a single order state equation.

$$x_s'' = v_m' = \frac{1}{m}(-k^* x_s - b v_m)$$

$$x_s'' = \frac{1}{m}(-k^* x_s - b x_s')$$

need in terms of x_s here.

2ND order state equation

$$\boxed{m x_s'' + b x_s' + (k_1 + k_2) x_s = 0}$$

This is the equation we need to solve. One way is to use Laplace transform. Another way is to use our "Guess" approach (actually these are equivalent).

Guess that $x_s(t) = A e^{rt}$

$$x_s' = r A e^{rt}$$

$$x_s'' = r^2 A e^{rt}$$

If this is the solution, must satisfy DEQ...

$$m(r^2 \cancel{Ae^{rt}}) + b(r\cancel{Ae^{rt}}) + (k_1+k_2)\cancel{Ae^{rt}} = 0$$

thus $\quad mr^2 + br + (k_1+k_2) = 0$

This is the <u>characteristic equation</u>.

To solve, need to find the values of 'r' that satisfy the characteristic equation. To find these, use <u>quadratic formula</u>

$$r_{1,2} = \frac{-b \pm \sqrt{b^2 - (4)(m)(k_1+k_2)}}{2m}$$

or rewrite,

$$\boxed{r_{1,2} = -\frac{b}{2m} \pm \sqrt{\left(\frac{b}{2m}\right)^2 - \frac{(k_1+k_2)}{m}}}$$

Thus the values of r_1, r_2 are dependent on the system parameters. Depending on these values, we'll get different system responses (see general vibrations book for more detail). Briefly, we have four cases...

For simplicity, consider a characteristic equation of the form

$$mr^2 + br + k = 0$$

$$r_{1,2} = -\frac{b}{2m} \pm \sqrt{\left(\frac{b}{2m}\right)^2 - \frac{k}{m}}$$

It's beyond the scope of this class, but the "damping ratio" ζ is defined as

$$\zeta = \frac{b}{2\sqrt{km}}$$

Thus there are four possibilities:

Case 1 - undamped. $b=0$, $\zeta = 0$

Case 2 - underdamped $\zeta < 1$

Case 3 - critically damped $\zeta = 1$

Case 4 - overdamped $\zeta > 1$.

(14)

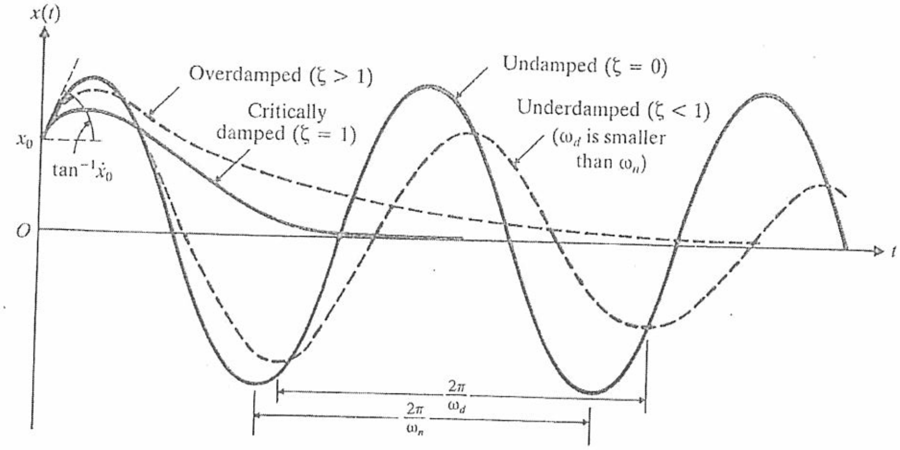


Figure 2.16. Comparison of motions with different types of damping.

"Mechanical Vibrations", 2nd Edition, S.S. Rao.
© 1990.