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A vibration energy harvesting device with bidirectional resonance frequency tunability

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Abstract
Vibration energy harvesting is an attractive technique for potential powering of wireless sensors and low power devices. While the technique can be employed to harvest energy from vibrations and vibrating structures, a general requirement independent of the energy transfer mechanism is that the vibration energy harvesting device operate in resonance at the excitation frequency. Most energy harvesting devices developed to date are single resonance frequency based, and while recent efforts have been made to broaden the frequency range of energy harvesting devices, what is lacking is a robust tunable energy harvesting technique. In this paper, the design and testing of a resonance frequency tunable energy harvesting device using a magnetic force technique is presented. This technique enabled resonance tuning to $\pm 20\%$ of the untuned resonant frequency. In particular, this magnetic-based approach enables either an increase or decrease in the tuned resonant frequency. A piezoelectric cantilever beam with a natural frequency of 26 Hz is used as the energy harvesting cantilever, which is successfully tuned over a frequency range of 22–32 Hz to enable a continuous power output 240–280 μW over the entire frequency range tested. A theoretical model using variable damping is presented, whose results agree closely with the experimental results. The magnetic force applied for resonance frequency tuning and its effect on damping and load resistance have been experimentally determined.

(Some figures in this article are in colour only in the electronic version)

1. Introduction

The emergence of low power and wireless devices has sought portable and long lasting energy sources. Particularly in applications where the replacement of batteries is unfeasible, self power generating devices are of interest. Wireless sensors with self powering ability have the potential to be employed in autonomous condition monitoring and wireless data transmitting and receiving, and vibration energy harvesting has the potential to be one such alternative to power these devices. It can be clearly interpreted as an efficient way of powering these wireless sensors because of the omnipresence of vibrations. Further, minimal maintenance and the ability to be employed in hostile and inaccessible environment make it a highly attractive powering source. Other energy harvesting techniques, such as solar cells and thermoelectrics, are limited by the need for the presence of sunlight and thermal gradients, respectively. The omnipresent vibration energy allows harvesting energy on a continuous basis to power these wireless sensors that can be placed in inaccessible and hostile locations.

Three mechanisms are commonly pursued for energy scavenging applications: electrostatic, electromagnetic, and piezoelectric. One of the earliest generic models of a vibration energy harvesting device was based on spring–mass–damper model, where the energy dissipated in the electrical damper is equivalent to the electrical energy generated [1]. Many researchers have attempted various techniques to harvest vibration energy efficiently and effectively. For example, an electrostatic-based energy harvesting device was developed using a variable capacitor, which was found to increase the corresponding voltage output by orders of magnitude [2].

In other work, a dynamic analysis was performed to enable non-resonant electrostatic energy harvesting for low frequency and high amplitude applications [3]. In addition, Despesse and co-workers have designed and modeled a high damping electrostatic vibration energy harvesting device to recover power over a large spectrum range below 100 Hz [4, 5]. Other efforts to maximize the power density via miniaturization of the energy harvesting device include fabrication of a millimeter scale device which is capable of harvesting 0.3 μW at 4 MHz [6]. Recently, an electrostatic vibration to electrical energy convertor was proposed that could be fabricated using MEMS technology [7]. In addition to these electrostatic methods, various electromagnetic techniques for power generation have been developed, such as miniature prototypes using two and four magnets which can generate power using ambient vibrations [8]. A MEMS-based fabrication technique has also been proposed, with a corresponding finite element analysis showing the design to be an efficient energy harvesting device [9]. In addition to simple cantilever-based models, a novel way of up-converting the frequency was proposed to achieve higher power output from the given vibrations using electromagnetic power generation in a manner compatible with MEMS scale technology [10]. However, it has been proposed that the piezoelectric technique is the most efficient way of harvesting vibration energy by Roundy [11]. For example, a theoretical model with experimental results was presented showing that the technique can generate enough energy to power a Pico radio [12]. Other designs include a non-uniform thickness piezoelectric cantilever beam, which would have higher strain and consequently higher power output for a given excitation [13]. Apart from these designs, a MEMS scale piezoelectric energy harvesting cantilever has been developed [14, 15] that can have increased power density and the ability to be fabricated as an on-chip power source. A preliminary study of the effect of vibration amplitude on the performance of a lead magnesium niobate-lead titanate (PMN-PT) single-crystal beam with interdigitated electrode pattern has also been performed [16].

While many efforts have sought to increase the power density of vibration energy harvesting devices, most designs are based on a single resonance frequency, thereby limiting their performance to a short range of frequencies. In order for these energy harvesting devices to be commercially viable for many applications, they have to operate over a wide range of frequency without sacrificing power output. This is feasible only if the device frequency matches the source frequency, enabling it to be in resonance to generate maximum power output over a suitable frequency range. Since environmental vibration characteristics (in particular frequency) are likely to change over time, there needs to be a mechanism to tune the energy harvesting device in order to alter its natural frequency to match the source frequency.

There are a number of techniques through which the resonance frequency of the device can be tuned. The easiest ways to change frequency of the device would be to alter the mass, length, or thickness of the vibrating structure; however, it would be challenging to alter these parameters while the device is operational. Earlier, an active tuning technique was demonstrated that applied an electrical input to a piezoelectric bimorph to alter the resonance frequency [17]. However, even though the technique allowed tuning of the resonance frequency, the continuous input power required for active tuning of the beam reduces the efficiency of the energy harvesting device. Recently, another technique was presented which tunes the resonance frequency of the device by the application of an axial compressive load. The applied load alters the stiffness of the simply supported beam, thereby altering the corresponding resonance frequency [18]. While this technique allowed for an efficient power output over an appreciable range of frequency, operability over a broader frequency range would be desirable. To overcome the need to employ a tuning technique, a multi-frequency piezoelectric energy harvester consisting of a plurality of cantilevered beams from a fixed base has been presented [19]. While successful in enabling appreciable power to be generated over a larger frequency range than is possible with a single cantilever beam, such an approach guarantees that a large subset of the beams will not be in resonance and hence will not contribute to the power generation at a given frequency, which drastically reduces the potential power density of such of a device. Recent efforts have also sought to include the development of a resonant energy MEMS array to increase the power density of the energy harvesting device [20]. Despite these advances, in terms of power density and efficiency over the frequency range of operation, enhancements are still necessary for a robust energy harvesting device methodology to be realized. In particular, while active techniques can be employed to tune the energy harvesting devices, one must of course consider the energy requirements of active tuning and the corresponding implications on the overall efficiency of the energy harvesting system.

In the present approach a magnetic force is used to alter the overall stiffness of the energy harvesting device. This enables one to increase or decrease the overall stiffness of the device using magnetic force to change the natural frequency of the device. Here attractive magnetic force is used to shift the beam natural frequency to lower frequencies with respect to the original resonant frequency (with no magnetic force present), while a repulsive magnetic force tunes to frequencies that are higher than the beams original natural frequency. While the tuning technique itself can be applied to any of the three energy harvesting techniques described earlier, in the current work the resonant frequency tunable energy harvesting device uses the piezoelectric technique for energy harvesting.

2. Theoretical background

2.1. Introduction

The device comprises a piezoelectric cantilever beam with a tungsten mass at its free end. By virtue of the piezoelectric property, the cantilever beam produces electrical energy when subjected to mechanical stress induced from vibrations. A tip mass is used to lower the natural frequency of the piezoelectric beam and increase the output power of the energy harvesting device. One potential application of such devices would be to harvest energy from low-level ambient environmental

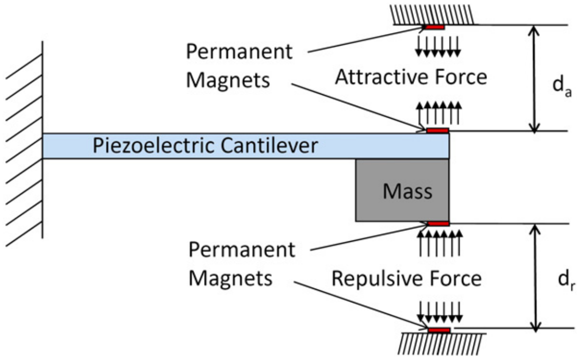

Figure 1. Schematic of the resonance frequency tunable energy harvesting device.

sources, which the literature suggests are typically under 100 Hz [13, 21]. For resonance frequency tuning, a magnetic force technique is proposed in which the applied magnetic force alters the effective stiffness of the device. Four permanent magnets are used: two magnets are fixed at the free end of the cantilever beam, while the other two magnets are fixed to the enclosure of the device at the top and bottom, vertically aligned with the magnets on the beam as depicted in figure 1. The magnets are placed such that attractive and repulsive magnetic forces can be applied on each side of the beam (the location of the attractive and repulsive magnetic forces can be interchanged with respect to their location). The cantilever beam is fixed on a clamp that can be vertically displaced using a screw–spring mechanism[1]. With this mechanism the distance between the magnets can be controlled to alter the magnetic force that exists between the magnets on the beam and enclosure. The magnetic force from the magnet induces an additional stiffness on the vibrating element which in turn alters the resonance frequency of the piezoelectric beam. The additional stiffness induced from the magnetic force is positive for repulsive force and is negative for the attractive force, respectively. It is noteworthy that by applying either an attractive or repulsive magnetic force, the natural frequency of the beam can be tuned to higher and lower frequencies with respect to the untuned resonance frequency of the piezoelectric beam.

2.2. Application of magnetic force

Cylindrical magnets are used to apply the desired magnetic force on the piezoelectric beam. The magnetic force between any two cylindrical magnets is given as

$$F_{\mathrm{mag}}(d) = \left[\frac{B_r^2 A_m^2 (l+r)^2}{\pi \mu_0 l^2}\right]\left[\frac{1}{d^2} + \frac{1}{(d+2l)^2} - \frac{2}{(d+l)^2}\right] \quad (1)$$

where B_r is the residual flux density of the magnet, A_m is the common area between the magnets, l is the length of the magnet, r is the radius of the magnet, d is the distance between the magnets, and μ_0 is the permeability of the intervening medium. Note that equation (1) is only an approximation which does not consider hysteresis between the magnetic force of attraction and repulsion. The associated values of the variables as used in the prototype described later in this paper are listed in table 1.

[1] The screw–spring mechanism consists of a screw inserted in a spring that is clamped on both ends. This enables the cantilever to be displaced vertically in either direction without in-plane rotation of the cantilever beam.

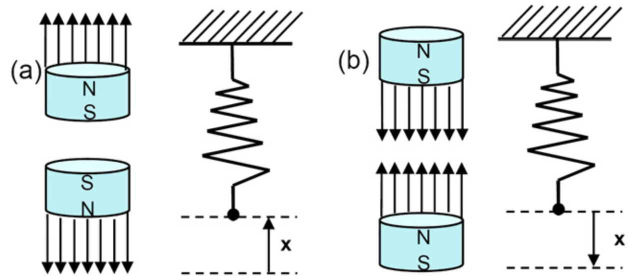

Figure 2. Spring equivalents for (a) repulsive force, resulting in increased stiffness, and (b) attractive force, resulting in decreased effective stiffness, of the energy harvesting device.

Table 1. Variable descriptions and their values.

Symbol	Description	Value	Units
B_r	Residual flux density	1.1	T
A	Area of the magnets	25.1	mm^2
L	Length of the magnet	2.95	mm
R	Radius of the magnet	4	mm
μ_o	Permeability of intervening medium	1.256×10^{-6}	H m^{-1}
A_m	Common area between magnets	25.1	mm^2
E	Young's modulus	3.81×10^{10}	GPa
I	Moment of inertia	0.36	mm^4
L	Length of the beam	34	mm
b	Width of the beam	20	mm
h	Thickness of the beam	0.6	mm
m_{eff}	Effective mass of the beam	45.8	g
$-d_{31}$	Piezoelectric strain coefficient of the material	-1.75×10^{-10}	C N^{-1}
ε	Dielectric constant	1.55×10^{-8}	F m^{-1}
C_p	Capacitance	1.7×10^{-7}	F
t_p	Thickness of piezoelectric layer	0.16	mm

It is evident from equation (1) that the magnetic force depends on the magnetic flux density, the common area between the magnets, and the distance separating the magnets. Any of the geometrical parameters can be altered to change the magnetic force between the two magnets. The common area between the magnets would be a difficult choice, as the area of the magnets used is very small, and it requires high precision to obtain the desired magnetic force. Hence the distance between the magnets is chosen to alter the magnetic force between the magnets. The attractive force can be represented as a spring in tension, and the repulsive force as a spring in compression, as shown in figure 2. Further, as described in section 2.4, the application of the magnetic force induces an additional stiffness within the device. An example of a plot of magnetic force and the corresponding magnetic stiffness versus distance, given the parameters in table 1, plotted in log–log scale, is given in figure 3.

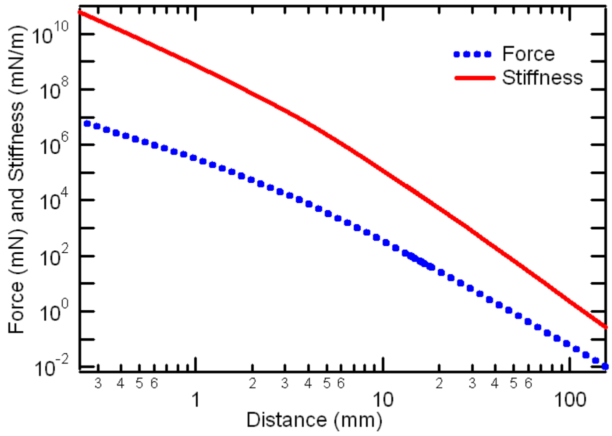

Figure 3. Plot of the force and stiffness versus separation distance between two cylindrical magnets in log–log scale.

2.3. Limitations of the application of magnetic forces

Using an applied magnetic force to alter the beam stiffness for tuning to different resonance frequencies has a limitation. In attractive mode, as the distance between the magnets decreases, the attractive magnetic force increases, and at a certain distance it will be larger than the stiffness force of the beam. In this case the magnets would snap together, changing the overall boundary conditions of the system [10]. The beam would then be subjected to fixed–fixed rather than fixed–free boundary conditions, thereby shifting the first resonant frequency of the beam far from the initial value. In order for the beam to operate in the designed frequency range, the magnets would then have to be separated, which would require a large force to overcome the force of attraction between the magnets. In this case the magnetic force of attraction needs to be smaller than the tip force of the beam at a given deflection to allow proper tuning of the beam without coming into contact, i.e.

$$F_{\mathrm{mag}} \leqslant K_{\mathrm{beam}} \cdot x \tag{2}$$

where K_{beam} is the stiffness of the beam (see equation (5)), and x is the tip deflection of the beam. The device is designed such that the magnets would not snap into contact.

For the repulsive magnetic force mode, if the magnetic force exceeds a certain force, the stresses induced in the beam would increase beyond the yield strength of the beam. Hence the stress resulting from the magnetic force (σ_{mag}) should be smaller than the yield stress of the beam (σ_{yield}), such that

$$\sigma_{\mathrm{mag}} \leqslant \sigma_{\mathrm{yield}} \tag{3}$$

where the stress due to the magnetic force is given as

$$\sigma_{\mathrm{mag}} = \frac{(M_{\mathrm{mag}})(c)}{I} = \frac{(F_{\mathrm{mag}}) \cdot (L) \cdot (h/2)}{I} \tag{4}$$

where I is the moment of inertia of the beam.

2.4. Effect of magnetic stiffness on resonance frequency

In the case of no magnetic force, the effective stiffness of a multilayered (through the thickness) cantilevered beam can be written as

$$K_{\mathrm{beam}} = \frac{b}{4L^3}\left(\sum_{i=1}^{n_1} n_i E_i h_i^3 + \sum_{j=1}^{n_2} n_j E_j h_j^3\right) \tag{5}$$

where b is the width of the beam, n_1 and n_2 are the numbers of piezoelectric and electrode layers, respectively, E_i and h_i are the Young's modulus and height of each piezoelectric layer, and E_j and h_j are the Young's modulus and height of each electrode layer. The effective mass of a multilayered cantilever beam with a tip mass can be approximated as

$$m_{\mathrm{eff}} = m_{\mathrm{t}} + 0.23bL\left(\sum_{i=1}^{n_1} n_i \rho_i h_i + \sum_{j=1}^{n_2} n_j \rho_j h_j\right) \tag{6}$$

where ρ_i and ρ_j are the densities of the piezoelectric and electrode materials and m_{t} is the tip mass. From this the corresponding natural frequency of the multilayered cantilevered beam is given as

$$\omega_{\mathrm{beam}} = \sqrt{\frac{K_{\mathrm{beam}}}{m_{\mathrm{eff}}}}. \tag{7}$$

The application of a magnetic force results is an additional stiffness that is introduced within the system, such that the resonance frequency of the device is now a function of both the beam stiffness and the stiffness associated with the magnetic force. The magnitude of the magnetic stiffness $\overline{K}_{\mathrm{mag}}$ can be written in terms of the change in magnetic force as a function of distance as

$$\overline{K}_{\mathrm{mag}} = \left|\frac{\delta F_{\mathrm{mag}}}{\delta d}\right| = \left|\frac{\partial F_{\mathrm{mag}}}{\partial d}\right| \tag{8}$$

where the change in force (and hence the magnetic stiffness) is positive for the repulsive magnetic mode and negative for the case of the attractive magnetic mode. From the magnetic force equation (1), the magnitude of the magnetic stiffness $\overline{K}_{\mathrm{mag}}$ is thus

$$\overline{K}_{\mathrm{mag}}(d) = \left|\frac{B_{\mathrm{r}}^2 A_{\mathrm{m}}^2 (l+r)^2}{\pi \mu_0 l^2} \times \left[-\frac{2}{d^3} - \frac{2}{(d+2l)^3} + \frac{4}{(d+l)^3}\right]\right| \tag{9}$$

where the sign of K_{mag} is dependent on the mode of the magnetic force (i.e. positive for repulsive mode and negative for attractive mode).

Given the additional stiffness introduced via the application of the magnetic force, the system can be represented using the lumped model shown in figure 4. Here the magnetic stiffness acts as a variable spring, whose stiffness is dependent on the change in magnetic force over the separation distance as shown in equation (8) for the cases of repulsive and attractive magnetic forces, respectively. Thus the total effective stiffness would be smaller or larger than the beam stiffness based on the mode of magnetic force induced and the sign of K_{mag}, such that

$$K_{\mathrm{eff}} = K_{\mathrm{beam}} + K_{\mathrm{mag}}. \tag{10}$$

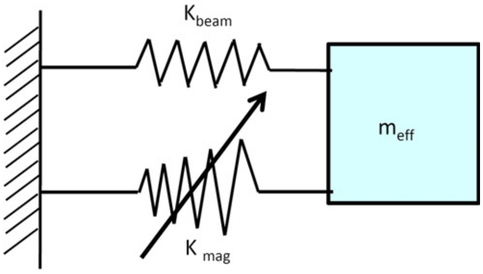

Figure 4. Lumped model of the device.

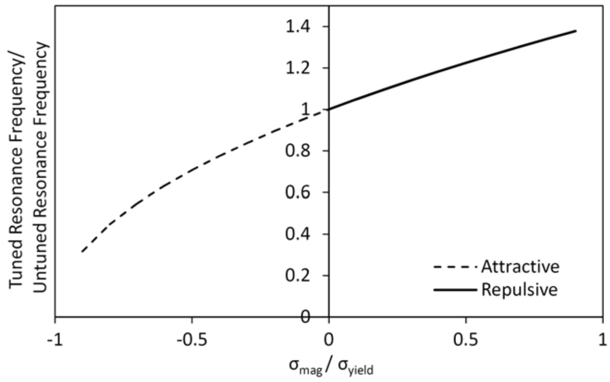

Figure 5. Resonance frequency versus stress resulting from the applied magnetic force (normalized with respect to the yield strength of the beam).

The effective stiffness K_{eff} would thus be substituted in equation (7) in place of K_{beam} to determine the resonance frequency of the energy harvesting device.

The maximum tunable frequency range that can be obtained in this case is limited by the yield strength of the beam (assuming that the magnets cannot come in contact for the case of the attractive magnetic mode). The plot in figure 5 shows the effect of the ratio of the stress due to the magnetic force on the beam (σ_{mag}) defined in equation (4) to the yield stress of the material of the beam (σ_{yield}) on the resonance frequency of the device. As the stress due to the magnetic force on the beam increases, the resonance frequency of the beam changes. As indicated above, the tuned resonance frequency is thus dependent on the mode of magnetic force (attractive and repulsive) and hence the resulting magnetic stiffness that is applied at the tip of the beam. As shown in figure 5, in repulsive mode the maximum tuned frequency is $\sqrt{2}$ times the untuned resonance frequency of the beam, after which the required magnetic force would exceed the yield strength of the beam. In attractive mode, the tuned natural frequency could theoretically approach zero, although in practice the device tuning to low frequencies is limited by the condition at which the magnets come into contact in attractive mode.

Given the limitations described above, one can substitute equation (7) into (10) to calculate the required applied magnetic stiffness to tune the vibrating beam from an original resonance frequency ω_{beam} to the desired tuned resonance frequency ω_{t}:

$$K_{\mathrm{mag}} = m_{\mathrm{eff}}\omega_{\mathrm{t}}^2 - K_{\mathrm{beam}} = m_{\mathrm{eff}}(\omega_{\mathrm{t}}^2 - \omega_{\mathrm{beam}}^2). \qquad (11)$$

2.5. Power output modeling of the cantilevered beam

The average effective stress per unit length induced in the vibrating beam subjected to a bending moment $M(x)$ is

$$\sigma_{\mathrm{beam}} = \frac{1}{L}\int_0^L \frac{M(x)c}{I}\,\mathrm{d}x \qquad (12)$$

where L is the length of the beam, c is the maximum distance from the neutral axis, I is the moment of inertia, and x is the direction along the length of the beam. The moment $M(x)$ considering the beam to be in resonance can be evaluated as

$$M(x) = K_{\mathrm{eff}} \cdot Y \cdot x = K_{\mathrm{eff}} \cdot \frac{1}{2\zeta_{\mathrm{t}}}\left(\frac{a}{\omega_{\mathrm{t}}^2}\right) \cdot x \qquad (13)$$

where Y is the tip deflection or amplitude of vibration also shown in equation (20), and ζ_{t} is the total damping in the system, which is determined using equation (19). Since the cantilevered beam is made of piezoelectric material, and assuming that the electrodes run the entire length of the beam, the voltage produced is related to the average effective stress in the beam as

$$V = \frac{-d_{31}t_{\mathrm{p}}\sigma_{\mathrm{beam}}}{\varepsilon} \qquad (14)$$

where t_{p} is the thickness of the piezoelectric layer, $-d_{31}$ is the piezoelectric strain constant and ε is the dielectric constant of the piezoelectric material. Since the piezoelectric material is a dielectric and is sandwiched between electrodes, it acts as a capacitor whose capacitance is given as

$$C_{\mathrm{p}} = \frac{n \cdot \varepsilon \cdot W \cdot L_{\mathrm{e}}}{t_{\mathrm{p}}} \qquad (15)$$

where n is the number of piezoelectric layers, W is the width of the piezoelectric layers, and L_{e} is the length of the electrode. The output power is given as

$$P = \frac{V^2 R_{\mathrm{L}}}{(R_{\mathrm{S}} + R_{\mathrm{L}})^2} \qquad (16)$$

where R_{S} is the impedance of the piezoelectric cantilever beam (also referred to as the source resistance), which depends on the frequency ω as follows:

$$R_{\mathrm{S}} = \frac{1}{\omega_{\mathrm{t}} \cdot C_{\mathrm{p}}}, \qquad (17)$$

and R_{L} is the load resistance. Note that when $R_{\mathrm{S}} = R_{\mathrm{L}}$, the device power output is maximum, a condition referred to as impedance matching. In this case the corresponding power is given as

$$P = \frac{V^2}{4R_{\mathrm{S}}}. \qquad (18)$$

3. Experimental method

The device is made up of a piezoelectric ceramic stripe actuator (APC International Ltd) used as a cantilever beam, which

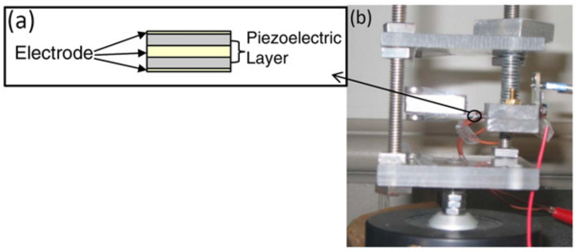

Figure 6. (a) Cross-section of the piezoelectric stripe actuator. (b) Photograph of the tunable energy harvesting prototype device.

consists of two piezoelectric layers sandwiched between three electrodes, as shown in figure 6(a). A tungsten mass is attached at the tip of the cantilevered beam to lower its natural frequency. Lead wires are soldered from the three electrodes of the piezoelectric beam for electrical output. The beam is fixed onto a clamp that can be vertically displaced using a screw–spring mechanism to change the distance between the end of the beam and the external magnets connected to the device housing. The screw–spring mechanism to displace the energy harvesting cantilever vertically is inserted in between the top of the enclosure and the clamp holding the beam. The beam can be moved up or down based on the rotation of the screw; the stiffness of the spring acts as a flexible clamp to enable fixture of the beam at the desired location. The pitch of the screw and the number of turns determines the distance moved in the axis of motion. The whole device is enclosed in a Plexiglas housing. Four permanent cylindrical magnets (Radio Shack) are used to tune the resonant frequency of the device. The magnets are placed as described in section 2.1. A photograph of the device is shown in figure 6(b).

The magnets are arranged such that attractive and repulsive forces exist on either side of the cantilever beam, with matching magnets fixed to the device. The built device is mounted on an electrodynamic mini-shaker (Bruel and Kjaer) and the lead wires from the piezoelectric beam are connected to the data acquisition (DAQ) card (National Instruments), which in turn is connected to the computer by a USB cable. An accelerometer (Analog Devices) is mounted on the clamp of the piezoelectric beam, which is also connected to the DAQ card to monitor the frequency and acceleration amplitude changes. LabView software is used for real time monitoring of voltages from the piezoelectric beam and accelerometer. The input frequency and acceleration are provided from a function generator (HP 4120 series) with a power amplifier (Bruel and Kjaer) connected to the shaker. The resonance frequency of the vibrating beam under a given set of conditions (i.e. applied magnetic stiffness, tip mass, etc) is found by noting the peak voltage output from the piezoelectric cantilever beam as a function of the excitation frequency externally controlled by the function generator. Once the resonance frequency is found, the excitation frequency is altered and the power output is monitored as a function of the applied magnetic force. In order to obtain precise force and displacement values between the magnets, the experiment has also been repeated by placing the fixed permanent magnets on the shaft of a dynamic mechanical analyzer (RSA III, TA Instruments, New Castle, Delaware), which has the ability to measure force and displacement with very high resolution. The complete layout of the experiment, detailing the electrical and mechanical connections involved, has been depicted in figure 7.

4. Results and discussion

4.1. Power output and resonant frequency

The power generated from the cantilevered beam has been monitored in real time using LabView software. A frequency of 26.2 Hz at 0.8 m s^{-2} acceleration is provided by the mini-shaker, which is the resonant frequency of the untuned beam (in the absence of an applied magnetic force). As the frequency is altered between 20 and 32 Hz, the power generated in the beam drops tremendously as the source frequency is shifted away from the initial resonant frequency of the beam. Next, the beam is tuned by inducing the required magnetic stiffness through the application of magnetic force by adjusting the distance between the magnets, which alters the beam resonance frequency to match the source frequency. The experimental results shown in figure 8 indicate that for this prototype the beam is successfully tuned between 22 and 32 Hz with the power harvested being in the range 240–280 μW. In figure 8 the thick solid line represents the power output that was successfully obtained for the case of the tuned energy harvesting system. Note that for the current device tuning to frequencies less than 22 Hz, using the attractive magnetic mode is limited by the condition that the magnets come into contact.

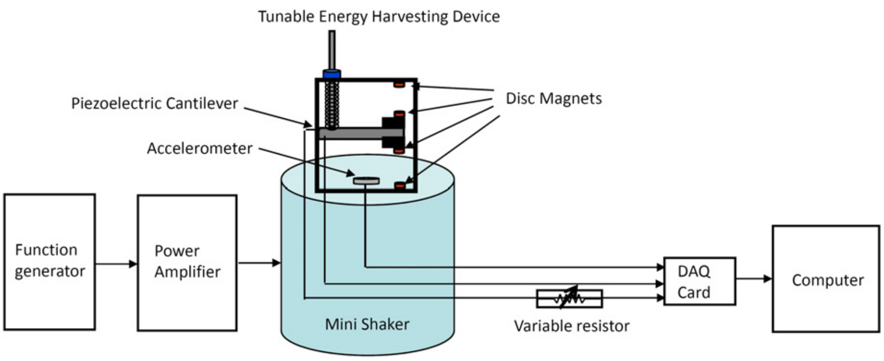

Figure 7. Layout of the experimental setup.

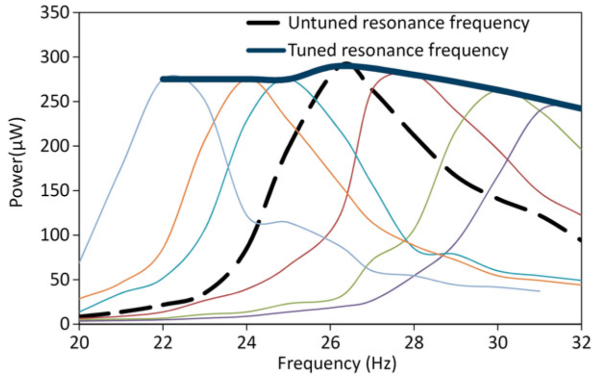

Figure 8. Experimental values of power output versus resonance frequency.

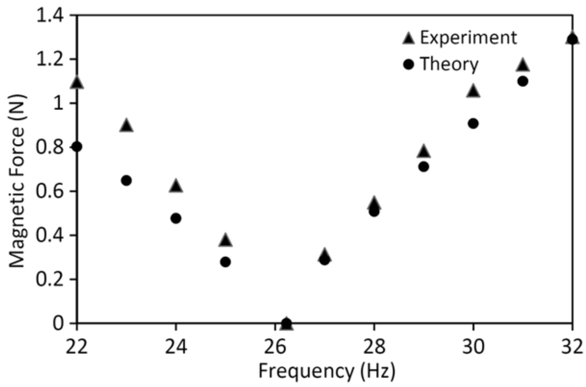

Figure 9. Experimental and theoretical values of applied magnetic force versus frequency necessary to tune the energy harvesting device.

4.2. Magnetic force versus resonance frequency

The magnetic force required to induce the magnetic stiffness necessary to tune the device for various resonance frequencies has been determined both theoretically and experimentally. The amount of magnetic stiffness required increases for tuning resonance frequencies that are far from the untuned resonance frequency. The theoretical value of the magnetic stiffness required to tune to the desired frequency is determined using equation (10), with the corresponding distance between the magnets determined from equation (9). Based on this separation distance, the applied magnetic force is found from equation (1). Experimentally, the distance between the magnets is manually determined for various resonance frequencies tuned. The magnetic force between the magnets at this distance is then measured by vertically aligning the magnets on the DMA at the same separation distance, which has the capability of reading the force and distance precisely. The theoretical and experimental magnetic forces required for tuning the device over the frequency range is shown in the following figure 9.

4.3. Damping and resonant frequency

The induced stiffness from the applied magnetic force also induces a certain amount of additional damping within the device. As shown in the next section, the assumption that damping is constant over the frequency range tested (and hence using the damping value determined for the untuned beam) does not accurately describe the trends in output power observed experimentally. It is observed that the damping increases with the increase in induced stiffness required for resonance frequency tuning. The damping of the piezoelectric beam increased from 0.042 to ~0.072 with the application of the applied magnetic force, as shown in figure 10. The damping is determined by first tuning the resonance frequency via the magnetic stiffness approach, and then performing a flick test to obtain an amplitude decay plot with time. From this amplitude decay plot the damping is calculated using the relationship

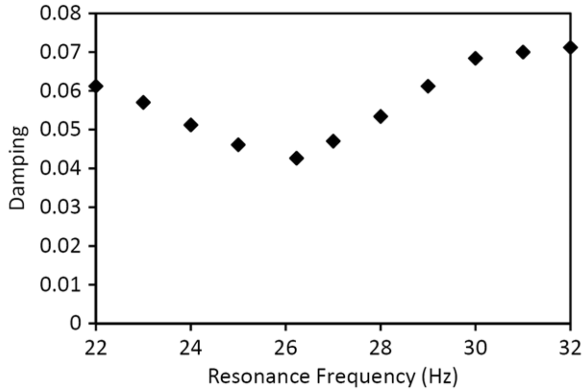

Figure 10. Experimentally measured values of damping versus resonance frequency.

$$\varsigma = \frac{1}{2\pi} \ln\left(\frac{a_1}{a_2}\right) \tag{19}$$

where a_1 and a_2 are consecutive amplitudes.

4.4. Power output (experimental versus theoretical)

The power generated from the tunable energy harvesting device measured experimentally can be compared to the theoretical model described in section 2. Two different models have been developed to match the experimental results obtained from tuning the device over the frequency range. Initially a constant damping model was developed, considering that the damping in the system would remain constant with the application of the magnetic stiffness. With this assumption, the output power decreases with the increase in resonant frequency. The drop in power is due to the decrease in the amplitude of vibration at higher frequencies. Here the amplitude Y is inversely proportional to the square of the frequency and is given as

$$Y = \frac{1}{2\zeta}\left(\frac{ma}{K_{\mathrm{eff}}}\right) = \frac{1}{2\zeta}\left(\frac{a}{\omega_{\mathrm{t}}^2}\right) \tag{20}$$

where a is the acceleration amplitude of the source, m is the mass of the beam, and ζ is the damping.

Based on the comparison of the initial experimental results with the model predictions assuming a constant damping over the range of frequencies tested, the initial model was modified to account for differences in damping values found at different frequencies (see section 4.3 and figure 10). The calculation

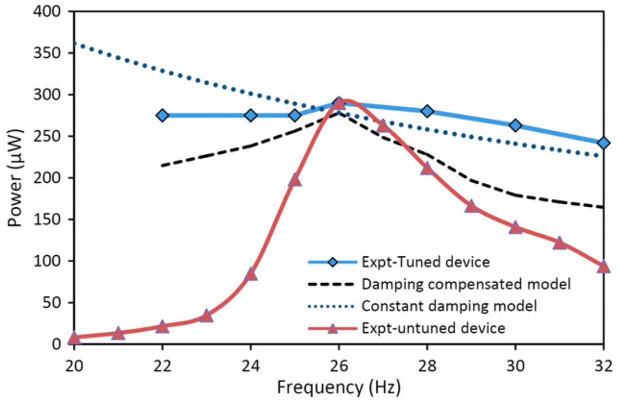

Figure 11. Comparison of experimentally measured power output with theoretical predictions using the constant damping and damping compensated models.

of the theoretical power output was thus modified using these experimental values of damping at each frequency, which is referred to as the damping compensated model. As shown in figure 11, the damping compensated model closely agrees with the experimental results. Also shown in figure 11 is the power output from the untuned energy harvesting device. By comparing the effective power output of the untuned beam with the tuned energy harvesting device, it is clear that the magnetic stiffness approach has been successful in enabling optimal energy harvesting of the device over the frequency range tested.

4.5. Optimal resistance

The power output generated has to be optimized with respect to load resistance to obtain maximum power at any given frequency. Here the resistance is altered to monitor the power output to determine the optimal resistance. The value of the resistance is noted down as the optimal resistance when the output power is maximum. Figure 12(a) shows the plot of the load resistance values to experimentally determine the optimal resistance of the beam with no applied magnetic field, which is found to be 26 kΩ. Similar experiments are performed at each frequency of the beam as it is tuned using the magnetic force induced stiffness to obtain the optimal resistance values. The source impedance is calculated theoretically using equation (17), and is compared with the experimental load resistances in figure 12(b). The discrepancy between theoretical and experimental load resistances is attributed to the fact that changing the load resistance alters the damping in the system, which in turn slightly alters the input vibration provided by the mini-shaker.

4.6. Bandwidth and power density of the device

The major advantage of the proposed magnetic force/stiffness tuning of the energy harvesting prototype is the 40% bandwidth that this solution provides. This bandwidth compares favorably with a value of 24% described earlier in the literature based on tuning the resonant frequency via the application of a compressive axial load to a simply supported vibrating beam [18]. Comparison of the energy/power densities achievable for different energy harvesting solutions is difficult due to the sensitivity of these values to the material selection, forcing frequency and acceleration amplitude, and the available bandwidths over which the device can harvest required levels of energy. However, some broad estimation of the power densities available for the different device designs provides some insight into the potential application of these various designs for a specific application.

An estimation of the working volume of the prototype device shown in figure 6 is approximately 50 cm^3. Figure 8 shows the power harvested from the device is 240–280 μW over the frequency range to which it is tuned, such that the energy density of the prototype is 5 $\mu\text{W cm}^{-3}$ operating at an acceleration amplitude 0.8 m s^{-2}. (We note that this level of acceleration is much smaller than that in many other references found in the literature, and that the power density is strongly dependent on the acceleration amplitude.) One can consider a 'competing' device to be an array of untuned cantilevers with resonant frequencies spanning the bandwidth obtainable using the proposed magnetic stiffness tuning approach. Considering a value of damping in the system to be 0.04 as measured for our system, an untuned cantilever has a bandwidth of 8% to achieve a minimum of 80% of peak power provided by the untuned cantilever operating at resonance. Thus for an energy harvesting device comprised of an untuned array of cantilevers, five cantilever beams would be required to provide a 40% frequency range. For a single cantilever a power density of

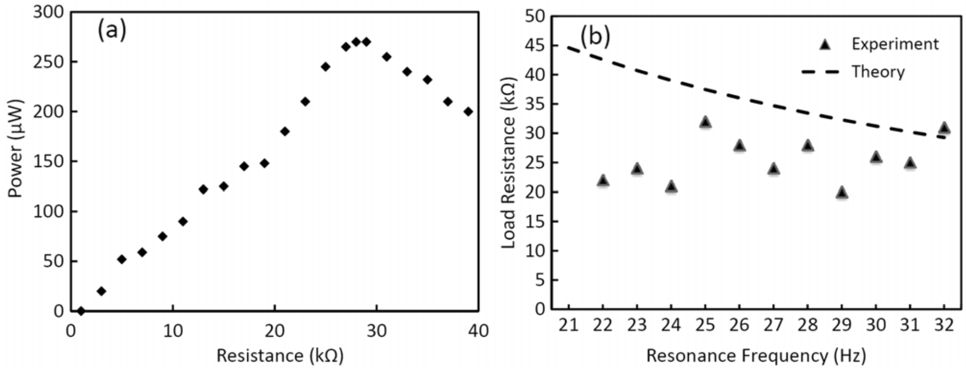

Figure 12. (a) Power versus load resistance at 26.2 Hz. (b) Load resistance versus resonance frequency.

375 μ W cm^{-3} was determined experimentally using a non-tunable, piezoelectric single cantilever driven at an acceleration amplitude of 2.25 m s^{-2} [22]. Five beams of various lengths are required to provide the bandwidth of the proposed magnetic tuning approach; such a device energy density would be 75 μW cm^{-3}.

In order to compare the energy density of the proposed magnetically tuned energy harvesting device, it is necessary to estimate the power output of the proposed device operating at this higher frequency and acceleration amplitude. Based on the fact that output power scales with the square of the acceleration amplitude, and adjusting the length of our beam to provide a frequency of 120 Hz, we conservatively estimate that the output power density of our device under identical operating conditions would be 75 μW cm^{-3}. While the values obtained in the conservative estimates are approximately equal, we note that the material used in [22] was a much higher quality piezoelectric material with a much larger coupling coefficient; this would result in much more efficient energy harvesting (note that the power density scales with the square of the coupling coefficient). In addition, our estimate for the energy density of the device comprised of an array of untuned cantilevers does not include the extra space necessary to separate the individual cantilevers (which hence adds to the volume of their device). In addition, for an array of untuned cantilevers evenly spaced throughout the required bandwidth, it cannot be guaranteed that any one cantilever will be at resonance (for example, the forcing frequency may fall exactly between the resonance frequency of two 'adjacent' cantilevers). With the proposed magnetic tuning approach one can ensure resonance of the energy harvesting device. For the device proposed in [18], where a compressive axial load is used to tune the resonance frequency to lower frequencies with respect to the untuned resonance frequency of the vibrating beam, it would be necessary to have two uniaxially compressed beams to match the frequency range obtained by the proposed magnetically tuned stiffness device, resulting in an effective power density of 50–60 μW cm^{-3}.

From this general analysis it is clear that the proposed magnetic tuning technique can provide power densities which are at least comparable to those which can be obtained using other techniques to provide a wide bandwidth energy harvesting device. Each of these techniques to increase the operational bandwidth of the harvesting device offers inherent advantages and disadvantages, and the appropriate method for a particular design application can only be chosen upon careful consideration of that application and optimization of the device design. However, the achievable bandwidth of 40% demonstrated using the magnetically tuned resonance technique provides additional design possibilities for developing energy harvesting devices.

5. Conclusions

Vibration energy harvesting is an attractive technique for the potential powering of wireless sensors and low power devices. A general requirement independent of the energy transfer mechanism is that the vibration energy harvesting device operate in resonance at the excitation frequency. Proposed within is a methodology to tune the resonant frequency of the vibrating structure within the energy harvesting device via the application of magnetic stiffness/forces via permanent magnets. By using either attractive or repulsive magnetic force, the resonant frequency can be shifted to either lower or higher frequencies in an effort to better match the source excitation frequency, making it bidirectionally tunable. In addition, it was found that damping increases as the magnitude of magnetic stiffness induced within the system increases. The distance between the magnets is limited by the geometrical stiffness in the beam and the magnetic flux density of the magnets. Magnetic contact can be avoided by optimizing the stiffness of the beam and the maximum distance between the magnets.

Using this technique an operable frequency bandwidth of 40% was achieved, such that a power of 240–280 μW could be obtained over the frequency range 22–32 Hz. While here the technique was demonstrated using a piezoelectric-based energy harvesting approach, the technique can readily be generalized to any of the techniques being pursued for energy harvesting applications. While the prototype device presented here has a frequency range 22–32 Hz, different frequency ranges could be targeted by designing and adjusting the resonant frequency of the untuned vibrating energy harvesting structure to fall within the targeted design frequency range, while an array of magnetically tuned energy harvesting cantilevers could clearly be used to provide an even broader operational frequency bandwidth (albeit at a decrease in power density). Further, utilizing the magnetic stiffness approach would enable an energy harvesting device to be employed in an alternative vibration environment characterized by a different forcing frequency (within the tunable range) or could offset the possibility that the built device has a resonant frequency that is different from the designed frequency and thus requires a tuning mechanism to maximize the energy output.

A conservative estimation of the power density (based on the operable bandwidth) achieved by the prototype device demonstrates that, despite the extra volume necessary to accommodate the permanent magnets necessary for the proposed tuning technique, the achievable power densities are comparable to those obtained using other methods. While the focus of this work was the modeling and experimental verification of the proposed resonant frequency tuning mechanism, it is to be noted that the prototype device described within is not completely optimized, and would possess higher power densities and bandwidth upon further optimization, which is currently under investigation.

The proposed technique is semi-active, which can be defined as a mechanism where an input energy is required only to adjust the separation distance between the magnets to a specific value; after the magnets have been displaced the device does not require additional input energy to keep the cantilever at that specific location due to the use of the screw–spring mechanism as described in section 2.1. From this standpoint the device is clearly more efficient than active tuning approaches that require constant power input to maintain the tuned frequency. Further, a simple calculation can be performed to estimate the amount of energy necessary to tune the device. Here the energy required to move the cantilever is the sum of the energy required to move the cantilever mass plus the energy required to offset the magnetic force through the distance the cantilever is moved. For the prototype device the energy required to move the cantilever (with tip mass) over a maximum distance of 3 cm (the maximum displacement from the neutral position of the cantilever) is approximately 85 mJ, assuming that an actuator with 60% efficiency is used. Given that the tuned device provides approximately 250 μW of power, one can estimate that it will take approximately 320 s to harvest sufficient energy from the tuned device to enable a tuning iteration.

Because the tuning mechanism is semi-active, the proposed technique may have potential applications in designing an autonomous energy harvesting device with self-tuning capability, where a feedback loop could control the applied magnetic force/stiffness and hence the resonant frequency of the device by altering the distance between the magnets to optimize the amount of harvested energy. This would enable the shifting of the resonance frequency of the energy harvesting system to account for changes in the characteristics of this peak amplitude of the forcing frequency that may occur over time. For example, the vibration characteristics of the beam and source can be determined through the power output and through an accelerometer, respectively. A mismatch in frequencies would provide information to the controller necessary to make a decision whether or not to adjust the mode or amount of magnetic force to be applied to the beam to tune to the given source frequency. Since an input energy is required to monitor the frequency changes and to move the magnets to the desired location, this task is performed only when there is a drop in output power. Once the device is tuned, the input power can be discontinued and the magnets would remain at the specified location until another tuning step is required. Because such a technique is semi-active, proper design of the proposed mechanism is an efficient means to optimize the output power for energy harvesting applications.

Of course, one must be careful that the energy cost of tuning the resonance frequency of the device does not exceed the level of energy harvested by the device or that is available to complete the tuning step. Certainly a case where the device is constantly ‘chasing’ the forcing frequency via continual altering of the applied magnetic stiffness would not be efficient. However, the development of the control logic and methodology for a ‘smart’ autonomous energy harvesting device which can alter its resonance frequency based on consideration of the total system efficiency (i.e. asking whether or not the increased harvesting performance obtained by a proposed tuning step outweighs the energy cost of performing the resonance tuning) is beyond the scope of this paper, and is the subject of future work in our group.

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