Joint Transmit and Receive Beamforming for Hybrid Active–Passive Radar

Yongchan Gao, Member, IEEE, Hongbin Li, Senior Member, IEEE, and Braham Himed, Fellow, IEEE

Abstract—We consider a new hybrid radar paradigm consisting of an active array and a passive array. In this hybrid system, the radar transmits a probing signal from its active array and receives two types of echoes: One is the return from the cooperative active transmission and the other is the target return due to transmission from noncooperative illuminators of opportunity. The motivation for this approach is to exploit not only the active signal but also any passive signals that are present in the surveillance area, thereby maximizing the signal-to-interference-plus-noise ratio (SINR). Numerical results demonstrate that the proposed concept can achieve a significant improvement of the output SINR, compared with conventional methods for active-only or passive-only radar systems.

Index Terms—Active signals, hybrid system, passive signals, receive beamforming, transmit beamforming.

I. INTRODUCTION

In recent years, there has been a growing interest in passive radar, which operates without transmitting its own signal but exploits wireless communication sources, referred to as noncooperative illuminators of opportunity (IOs) [1]. With the advances of wireless communication technologies and infrastructures, abundant IOs are available, including analog FM radio and television stations, digital audio/video broadcasting, GPS, WiFi signals, etc. Passive radar can simultaneously access several IOs at different locations and enjoy the associated spatial and frequency diversity. Passive radar is also far more economical compared with its active counterpart since there is no transmitter related cost [2], [3]. Despite such advantages, passive radar loses several fundamental attributes of active radar, such as the ability to use waveforms specifically designed for radar functions, flexible beam steering on transmit, higher detection range, and others.

In this letter, we propose a new paradigm of hybrid radar that consists of an active array and a passive array. The active array probes the environment with its own radar waveform and is able to perform transmit beamforming, whereas the passive array does not transmit but listens to target echoes from predetermined IO sources. Effectively, this forms a distributed multiple input multiple output (MIMO) radar systems that consists of an active MIMO subsystem with colocated antennas [4], [5] and several bistatic single-input multioutput radars formed by the spatially distributed IO sources and the passive arrays. The motivation of this letter is to benefit from several useful features of both active and passive radar, including but not limited to active beam steering of the former as well the economies and flexibility in forming a distributed sensing system and the associated diversities.

Of particular interest to this letter is beamforming for such a hybrid system. The problem of joint transmit and receive beamforming was investigated for active MIMO radar. Specifically, a design of the transmit beamforming correlation matrix and receive beamforming vector are studied in [6] by maximizing the output signal-to-interference-plus-noise ratio (SINR). Liu et al. introduced a joint transmit and receive beamforming in the presence of signal-dependent interference by sequential optimization [7]. An iterative method was proposed to jointly design the transmit signals and the receive filter in [8], where a transmit weighting matrix was used. Robust design for transmit waveform and receive filter was addressed in [9], which is solved by a cyclic algorithm.

We consider herein joint transmit and receive beamforming in a hybrid active–passive system, where the radar transmits active signals to the target and the array receives both the cooperative active and noncooperative passive signals. In this hybrid system, a joint transmit and receive beamforming approach aimed at maximizing the output SINR in the presence of signal-dependent interference is proposed. The advantage of utilizing both cooperative and noncooperative signals is demonstrated through numerical results. During the review process, [10] was brought to our attention. While [10] also considered a hybrid active–passive setup as in our letter, it focused on the fusion of the local detections made by each individual radar, based on standard fusion rules. In contrast, we consider joint transmit and receive optimization for the hybrid system by exploiting prior knowledge of the sensing environment, which is distinctively different from their problem.

Notations: $(\cdot)^*$, $(\cdot)^T$, and $(\cdot)^H$ denote the conjugate, transpose, and conjugate transpose, respectively. $E(\cdot)$ indicates the statistical expectation. $\|\cdot\|$ denotes the Euclidean norm.

II. SIGNAL MODEL

Fig. 1 depicts a hybrid active–passive radar system. The radar comprises both an active array, which transmits a probing waveform, and a passive array, which does not transmit but uses the waveform from an IO for target detection. As in standard passive radar, the system is equipped with a reference channel (RC), which utilizes a directional antenna pointed to the IO, to obtain
a copy of the IO waveform to serve as a reference. As shown in the right diagram of Fig. 1, the target, which is observed by both the active and passive arrays, is located at an intersection of a circle, which indicates the range of the active array, and an ellipse, which is the isorange of the passive array. In addition to the target, there may exist other scatterers within the same range or isorange, which act as interference signals in both the active and passive arrays.

Assume the active array consists of \( N_a \) collocated antennas. The baseband equivalent model of the transmitted signals from the active array is given by

\[
 t_s_a(l), \quad l = 1, \ldots, L_a
\]  

(1)

where \( t = [t_1, t_2, \ldots, t_{N_a}]^T \) is the beamforming vector designed to transmit a waveform \( s_a(l) \), and \( \|t\| = 1 \). Here, \( L_a \) denotes the number of samples in the waveform and \( t_n \) denotes the weight on the \( n \)th element. In the far field, the signal seen at the look angle \( \theta \) is considered as a superposition of the delayed and attenuated version of the transmitted signals. A narrowband system is assumed such that the signal at that location is expressed as

\[
 a^T_\theta(t) t_s_a(l)
\]  

(2)

where \( a_\theta(t) \) is the transmit steering vector. Assume further that there is a target located at \( \theta_0 \), along with multiple interference scattering sources located at \( \theta_i \neq \theta_0 \). The baseband equivalent of the signals from the \( N_a \) collocated receive elements is given by

\[
x_a(l) = \alpha_0 a_\theta(\theta_0) a^T_\theta(\theta_0) t s(l) + \sum_i \alpha_i a_\theta(\theta_i) a^T_\theta(\theta_i) t s_a(l) + n_a(l)
\]  

(3)

where \( a_\theta(\theta) \) denotes the propagation vector due to the propagation delays from a source to the receive elements, which is the same as the transmit steering vector, assuming the same array is used for both transmit and receive. \( n_a(l) \) denotes the additive white Gaussian noise vector, \( \alpha_0 \) and \( \alpha_i \) denote the complex amplitudes of the target and the \( i \)th interference source, respectively. The received signal is processed by a matched filter aligned to the target delay, which outputs \( y_a \)

\[
y_a = \frac{\sum_{l=1}^{L_a} x_a(l) s^*_a(l)}{|s_a(l)|^2}
\]

\[
= \alpha_0 a_\theta(\theta_0) a^T_\theta(\theta_0) t + \sum_i \alpha_i a_\theta(\theta_i) a^T_\theta(\theta_i) t + n'_a
\]

\[
= \alpha_0 A(\theta_0) t + \sum_i \alpha_i A(\theta_i) t + n'_a
\]  

(4)

where \( A(\theta) = a_\theta(\theta) a^T_\theta(\theta) \) and \( n'_a \) has zero mean and covariance matrix \( \sigma^2_n I \). The amplitudes of the target (for \( i = 0 \)) and the \( i \)th interference source (for \( i \neq 0 \)) are given by

\[
\alpha_i = \frac{\rho_i \sqrt{P_i}}{R^2}
\]

(5)

where \( \rho_i \) is a parameter determined by the radar cross section (RCS) of the \( i \)th source and channel-induced phase shift, \( P_i \) denotes the radar transmit power, and \( R \) denotes the range (source-to-receiver), namely, the radius of the circle in Fig. 1.

On the other hand, the passive array, which consists of \( N_p \) receive antennas, receives the echoes from potential targets due to the transmission from a noncooperative IO. The received signal consists of reflections from the target as well as interference scatterers, which are located on the same isorange as the target. The isorange is an ellipse whose foci coincide with the IO and passive receiver locations. Suppose the direct signal from the IO has been removed by some spatial/temporal interference cancelation schemes \([11]\). The signal received by the passive array can be written as

\[
x_p(l) = \gamma_0 a_p(\theta_0) s_p(l) + \sum_i \gamma_i a_p(\theta_i) s_p(l) + n_p(l)
\]  

(6)

\[
l = 1, \ldots, L_p
\]

where \( a_p(\theta) \) is the propagation vector, \( n_p(l) \) is the additive white Gaussian noise, and \( s_p(l) \) is the waveform transmitted by the IO, which is obtained from the RC. Similarly, \( \gamma_i \) in (6) is the scaling coefficient of the \( i \)th scattering source, which accounts for its reflectivity, the channel propagation, and the antenna gain. It can be expressed as

\[
\gamma_i = \frac{\xi_i \sqrt{P_p}}{R_{i,r,i}}
\]  

(7)

where \( \xi_i \) is the RCS of the \( i \)th scattering source, \( P_p \) is the transmit power from the IO, \( R_{i,r,i} \) are the transmit range (IO-to-the \( i \)th source) and the receive range (the \( i \)th source-to-receiver), respectively. Note that \( R_{r,0} \) for the target is the radius of the circle in Fig. 1, i.e., \( R_{r,0} = R \).

The reference signal observed from the RC is given by

\[
x_{pr}(l) = s_p(l) + n_p(l).
\]  

(8)

A copy of \( s_p(l) \) is obtained from an RC. By using a cross correlator \([11]\) that cross correlates the reference signal with the surveillance observation (6), the waveform \( s_p(l) \) is removed. Therefore, we have

\[
y_p = \gamma_0 a_p(\theta_0) + \sum_i \gamma_i a_p(\theta_i) + n'_p
\]  

(9)
where the noise $n_i^o$ has zero mean and covariance matrix $\sigma^2_n I$.

Hence, the received signal in the hybrid system is given by

$$y = \begin{bmatrix} y_0^T & y_p^T \end{bmatrix}^T. \tag{10}$$

The received signal is filtered through the weight vector below

$$w = \begin{bmatrix} w_0^T, & w_p^T \end{bmatrix}^T \in \mathbb{C}^{(N_a + N_p) \times 1}. \tag{11}$$

In this letter, we assume the active and passive arrays use different frequency bands, the output signals are given by $w_0^H y_a$ and $w_p^H y_p$, respectively. Then, the total output SINR can be expressed as

$$\text{SINR}(t, w) = \text{SINR}_a(t, w_a) + \text{SINR}_p(w_p) \tag{12}$$

where

$$\text{SINR}_a(t, w_a) = \frac{\sigma^2_n |w_0^H A(\theta_0) t|^2}{w_0^H R_a w_a + \sigma^2_n w_0^H w_a} \tag{13}$$

$$\text{SINR}_p(w_p) = \frac{\sigma^2_n |w_p^H A_p(\theta) t|^2}{w_p^H R_p w_p + \sigma^2_n w_p^H w_p}. \tag{14}$$

Here, $R_a = E(c_a c_a^H)$ with $c_a = \sum \alpha_i A(\theta_i)$ and $R_p = E(c_p c_p^H)$ with $c_p = \sum \gamma_i a_p(\theta_i)$. In order to maximize the output SINR, the joint design of the transmit beamformer $t$ and the receive beamformer $w$, can be formulated as the following optimization problem:

$$\begin{cases} \max_{t, w} \text{SINR}(t, w) \\ \text{s.t.} \|t\| = 1. \end{cases} \tag{15}$$

III. TRANSMIT AND RECEIVE BEAMFORMING

To solve (15), we should solve $w_a$, $t$, and $w_p$ separately for the different frequency bands of the active and passive radar. Similar to [7], [12], and [13], we first employ sequential optimization to solve $w_a$ and $t$, i.e., $w_a$ is first optimized for a fixed $t$ and then $t$ is optimized for a fixed $w_a$. Specifically, the optimization problem of solving $w_a$ for a given $t$ can be written as

$$\max_{w_a} \frac{\sigma^2_n |w_0^H A(\theta_0) t|^2}{w_0^H (R_a + \sigma^2_n I) w_a}. \tag{16}$$

The optimization problem in (16) can be recast as the well-known minimum variance distortionless response (MVDR) problem [14], namely

$$\begin{cases} \min_{w} w^H (R_a + \sigma^2_n I) w \\ \text{s.t.} \ w^H A(\theta_0) t = 1. \end{cases} \tag{17}$$

It follows from (17) that

$$w_a = \frac{(R_a + \sigma^2_n I)^{-1} A(\theta_0) t}{(R_a + \sigma^2_n I)^{-1} A(\theta_0) A(\theta_0)^{\dagger} t}.$$ \tag{18}$$

Now, we optimize $t$ in terms of $w_a$. For a fixed $w_a$, the optimal transmit beamformer can be obtained by solving the following optimization problem:

$$\begin{cases} \max_{t} \frac{\sigma^2_n |w_a^H A(\theta_0) t|^2}{t^H (R_w + \sigma^2_n w_a^H w_a I) t} \\ \text{s.t.} \|t\| = 1 \end{cases} \tag{19}$$

where $R_w = E(c_a w_a^H c_a^H)$. Then, (19) can be recasted as

$$\begin{cases} \min_{t} t^H (R_w + \sigma^2_n w_a^H w_a I) t \\ \text{s.t.} \|t\| = 1 \end{cases} \tag{20}$$

which yields

$$w_p = \frac{R_p + \sigma^2_n I - 1}{a_p(\theta_0)^{\dagger} (R_p + \sigma^2_n I)^{-1} a_p(\theta_0)}.$$ \tag{21}$$

Similarly, the optimal $t$ for (20) is given by

$$t = \varepsilon (R_w + \sigma^2_n w_a^H w_a I)^{-1} A(\theta_0) w_a,$$

where $\varepsilon$ is a scalar to ensure the unit norm of $t$. Note that the above sequential optimization algorithm is stopped when the SINR improvement is no more than a preassigned value (e.g., $\delta = 10^{-6}$).

Then, we solve the following problem to get $w_p$:

$$\begin{cases} \max_{w_p} \frac{\gamma_i |w_p^H a_p(\theta_0)|^2}{w_p^H (R_p + \sigma^2_n I)w_p} \end{cases} \tag{22}$$

which yields

$$w_p = \frac{R_p + \sigma^2_n I - 1}{a_p(\theta_0)^{\dagger} (R_p + \sigma^2_n I)^{-1} a_p(\theta_0)}.$$ \tag{23}$$

The above procedure for the optimization of both the active array and passive array is summarized in Table I.
IV. SIMULATION RESULTS

We present numerical results to demonstrate the proposed beamforming approach for the hybrid active-passive radar (referred to as Hybrid TxRx beamforming). It is compared with two beamformers in the active-only system, i.e., MVDR method with only receive beamforming [15] (referred to as Active Rx beamforming), and the joint transmit and receive beamformer (referred to as Active TxRx beamforming) proposed in [7].

First, we consider a scenario, where the transmitter and receiver of the active system share a uniform linear array (ULA) of \( N_a = 4 \) elements with a half-wavelength spacing, the passive array is also a ULA of \( N_p \) antennas, and a noncooperative IO is present. The scene of Fig. 1 is considered. The parameters associated with these sources are: \( P_a = P_p = 1 \), \( R = 3.5 \) km, \( R_{i,0} = 1.8 \) km, \( R_{p,1} = 3.3 \) km, \( R_{t,1} = 3.0 \) km, the direction of arrival (DOA) of the target is 30°. We consider one interference signal with DOA 20° (close to the target), and two other interference signals with DOAs 80° and 55° in the active part (i.e., the three interferences are on the circle), and the power of each interference is identical. Additionally, there is one interference signal with DOA -30° in the passive section (i.e., one interference is on the ellipses), and its power is determined by the square of (7). The interference-to-noise ratio (INR) and signal-to-noise ratio (SNR) are defined as

\[
\text{INR} = 10\log_{10} \frac{\sigma_T^2}{\sigma_n^2}
\]

(24)

and

\[
\text{SNR} = 10\log_{10} \frac{\sigma_T^2}{\sigma_n^2}
\]

(25)

For comparison purposes, both the INR and the SNR are for the active section, since traditional methods can only benefit from active-only signals. In the following simulations, we perform the proposed algorithm for 200 different random initializations of the transmit beamformer, and then choose the solution that corresponds to the maximum output SINR.

Fig. 2 shows the SINR behavior for all methods as a function of SNR for \( \text{SNR} = 20 \) dB. It is seen that the SINR of the Active-TxRx beamforming is higher than that of the Active-Rx beamforming. However, the Hybrid-TxRx beamforming achieves the highest SINR compared with other beamformers, since it uses both the active and passive signals for beamforming. Furthermore, the performance of the proposed approach can be improved even when the interference signal is close to the target.

Next, we consider a more general scenario, which is an urban environment with multiple passive sources as well as more interfering scatterers. The hybrid active–passive radar scene is shown in Fig. 3(a), where the circle corresponds to the TxRx pair for the active case and the three ellipses correspond to the TxRx pairs for the passive case. The transmitter and receiver share a ULA of \( N_a = 12 \) elements with a half-wavelength spacing and the transmit power of the active array is \( P_a = 1 \) unit. There are three IOs with \( P_{p,1} = 3 \), \( P_{p,2} = 4 \), and \( P_{p,3} = 1 \), respectively, in the surveillance area, and we further assume they all use \( N_p = 12 \) antennas. The powers of two IOs are higher since they are TV/broadcasting stations, for example. The receive range (target source-to-receiver) is \( R = 4.0 \) km, and the transmit ranges (IO-to-target source) are \( R_{t,1} = 1.6 \) km, \( R_{t,2} = 3.0 \) km, and \( R_{t,3} = 2.0 \) km, respectively. A number of 200 interference signals randomly distributed in the area, and located on the circle or ellipses of the hybrid system. To determine whether the \( i \)th interference is located on the \( j \)th isoranger, we compare the bistatic range \( R_j \) of the target associated with the \( j \)th and the bistatic range \( R'_j \) of the \( i \)th interference[16] \( |R'_j - R_j| \leq \frac{c}{2B} \), where \( c \) denotes the speed of light and \( B \) the bandwidth of the radar. In our simulation, we have \( B = 3 \) MHz. The DOA of the target is 228.6°. The number of interference signals on the active circle is more than that of the passive ellipses since the active transmitter is more easily affected by interference sources.

Fig. 3(b) shows the SINR performance as a function of SNR for INR = 20 dB. It can be seen that the proposed algorithm achieves the higher SINR compared with other methods. The proposed algorithm not only uses the active signals for beamforming, but also exploits the passive signals that are present in the surveillance area.

V. CONCLUSION

We have proposed a joint transmit and receive beamforming approach for hybrid active–passive radar. The merit of this approach is to exploit the benefits of using both cooperative active probing and noncooperative passive signals in the surveillance area for improving the radar performance. Numerical results have shown that the proposed method is able to achieve a considerable SINR improvement over traditional designs. The price paid for the gain is a higher cost/complexity incurred by the proposed hybrid system than either its active-only or passive-only counterpart.
REFERENCES


